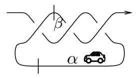
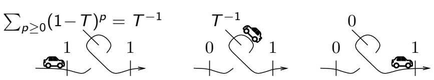


Definition. The traffic function $G = (g_{\alpha\beta})$ (also, the Green function or the two-point function) is the reading of a traffic counter at β, if car traffic is injected at α (if $\alpha = \beta$, the counter is after the injection point). There are also model-T_ν traffic functions $G_\nu = (g_{\nu\alpha\beta})$ for $\nu = 1, 2, 3$.

Example.



$$G = \begin{pmatrix} 1 & T^{-1} & 1 \\ 0 & T^{-1} & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

ωεβ:=http://drorbn.net/ktc25

Given crossings $c = (s, i, j)$, $c_0 = (s_0, i_0, j_0)$, and $c_1 = (s_1, i_1, j_1)$, let

$$\begin{aligned} F_1(c) = s\,[&1/2 - g_{3ii} + T_2^s g_{1ii} g_{2ji} - T_2^s g_{3jj} g_{2ji} - (T_2^s - 1) g_{3ii} g_{2ji} \\ &+ (T_3^s - 1) g_{2ji} g_{3ji} - g_{1ii} g_{2jj} + 2 g_{3ii} g_{2jj} + g_{1ii} g_{3jj} - g_{2ii} g_{3jj}] \\ &+ \frac{s}{T_2^s - 1}\,[(T_1^s - 1)\,T_2^s\,(g_{3jj} g_{1ji} - g_{2jj} g_{1ji} + T_2^s g_{1ji} g_{2ji}) \\ &+ (T_3^s - 1)\,(g_{3ji} - T_2^s g_{1ii} g_{3ji} + g_{2ij} g_{3ji} + (T_2^s - 2) g_{2jj} g_{3ji}) \\ &- (T_1^s - 1)(T_2^s + 1)(T_3^s - 1) g_{1ji} g_{3ji}] \end{aligned}$$

$$F_2(c_0, c_1) = \frac{s_1 (T_1^{s_0} - 1)(T_3^{s_1} - 1) g_{1 j_1 i_0} g_{3 j_0 i_1}}{T_2^{s_1} - 1} \left(T_2^{s_0} g_{2 i_1 i_0} + g_{2 j_1 j_0} - T_2^{s_0} g_{2 j_1 i_0} - g_{2 i_1 j_0}\right)$$

$$F_3(\varphi_k, k) = \varphi_k (g_{3kk} - 1/2)$$

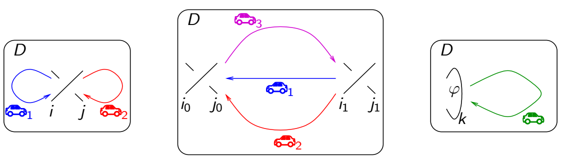
(Computers don't care!)

ωεβ:=http://drorbn.net/ktc25

Main Theorem.

The following is a knot invariant: (the Δ_ν are normalizations discussed later)

$$\theta(D) := \Delta_1 \Delta_2 \Delta_3 \left(\sum_c F_1(c) + \sum_{c_0, c_1} F_2(c_0, c_1) + \sum_k F_3(\varphi_k, k) \right).$$

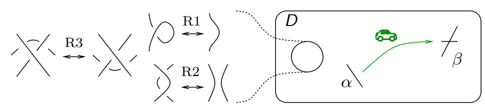


If these pictures remind you of Feynman diagrams, it's because they are Feynman diagrams [BN2].

ωεβ:=http://drorbn.net/ktc25

Lemma 1.

The traffic function $g_{\alpha\beta}$ is a "relative invariant":



(There is some small print for R1 and R2 which change the numbering of the edges and sometimes collapse a pair of edges into one)

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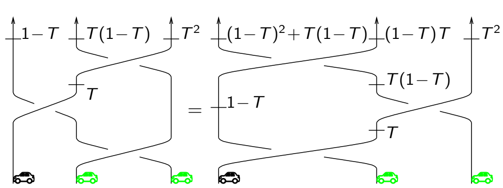
Proof.



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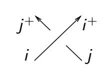
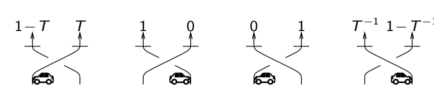
Lemma 2.



With $k^+ := k + 1$, the "g-rules" hold near a crossing $c = (s, i, j)$:

$$g_{j\beta} = g_{j^+\beta} + \delta_{j\beta} \quad g_{i\beta} = T^s g_{i^+\beta} + (1 - T^s) g_{j^+\beta} + \delta_{i\beta} \quad g_{2n^+,\beta} = \delta_{2n^+,\beta}$$

$$g_{\alpha i^+} = T^s g_{\alpha i} + \delta_{\alpha i^+} \quad g_{\alpha j^+} = g_{\alpha j} + (1 - T^s) g_{\alpha i} + \delta_{\alpha j^+} \quad g_{\alpha,1} = \delta_{\alpha,1}$$



ωεβ:=http://drorbn.net/ktc25

Corollary 1.

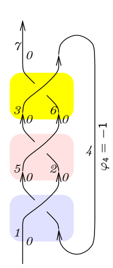
G is easily computable, for $AG = I$ $(= GA)$, with A the $(2n+1) \times (2n+1)$ identity matrix with additional contributions:

$c = (s, i, j) \mapsto$

A	col i^+	col j^+
row i	$-T^s$	$T^s - 1$
row j	0	-1

For the trefoil example, we have:

$$A = \begin{pmatrix} 1 & -T & 0 & 0 & T-1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -T & 0 & 0 & T-1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & T-1 & 0 & 1 & -T & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$



ωεβ:=http://drorbn.net/ktc25

And so,

$$G = \begin{pmatrix} 1 & T & 1 & T & 1 & T & 1 \\ 0 & 1 & \frac{1}{T^2-T+1} & \frac{T}{T^2-T+1} & \frac{T}{T^2-T+1} & \frac{T^2}{T^2-T+1} & 1 \\ 0 & 0 & \frac{1}{T^2-T+1} & \frac{T}{T^2-T+1} & \frac{T}{T^2-T+1} & \frac{T^2}{T^2-T+1} & 1 \\ 0 & 0 & \frac{1-T}{T^2-T+1} & \frac{1}{T^2-T+1} & \frac{1}{T^2-T+1} & \frac{T}{T^2-T+1} & 1 \\ 0 & 0 & \frac{1-T}{T^2-T+1} & -\frac{(T-1)T}{T^2-T+1} & \frac{1}{T^2-T+1} & \frac{T}{T^2-T+1} & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$