Question 8.

Is there a direct quantum field theory derivation of θ? Perhaps using the ϵ-expansion (at constant k!) of Chern-Simons-Witten theory with gauge group $\mathfrak{g}^{\epsilon}_{+} := \mathcal{D}(\mathfrak{b}, b, \epsilon\delta)$ with some Seifert-surface-dependent gauge fixing?

Expectation 9.

There are many further invariants like θ, given by Green function formulas and/or Gaussian integration formulas. One or two of them may be stronger than θ and as computable.

Dream 10.

These invariants can be explained by something less foreign than semisimple Lie algebras.

Dream 11.

θ will have something to say about ribbon knots.

Thank You!

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