



The Strongest Genuinely Computable Knot Invariant in 2024

Abstract. “Genuinely computable” means we have computed it for random knots with over 300 crossings. “Strongest” means it separates prime knots with up to 15 crossings better than the less-computable HOMFLY-PT and Khovanov homology taken together. And hey, it’s also meaningful and fun.



van der Veen

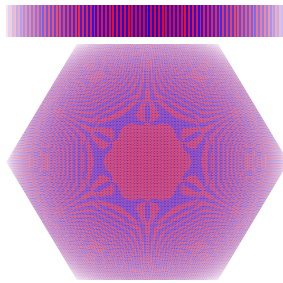
Continues Rozansky, Garoufalidis, Krickler, and Ohtsuki, joint with van der Veen.

Acknowledgement. This work was supported by NSERC grant RGPIN-2018-04350 and by the Chu Family Foundation (NYC).

Strongest. Testing $\Theta = (\Delta, \theta)$ on prime knots up to mirrors and reversals, counting the number of distinct values (with deficits in parenthesis):

	knots	(H, Kh)	(Δ, ρ_1)	$\Theta = (\Delta, \theta)$	together
reign		2005-22	2022-24	2024-	
xing ≤ 10	249	248 (1)	249 (0)	249 (0)	249 (0)
xing ≤ 11	801	771 (30)	787 (14)	798 (3)	798 (3)
xing ≤ 12	2,977	(214)	(95)	(19)	(18)
xing ≤ 13	12,965	(1,771)	(959)	(194)	(185)
xing ≤ 14	59,937	(10,788)	(6,253)	(1,118)	(1,062)
xing ≤ 15	313,230	(70,245)	(42,914)	(6,758)	(6,555)

Genuinely Computable. Here’s Θ on a random 300 crossing knot (from [DHOEBL]). For almost every other invariant, that’s science fiction.



Fun. There’s so much more to see in 2D pictures than in 1D ones! Yet almost nothing of the patterns you see we know how to prove. We’ll have fun with that over the next few years. Would you join?

Meaningful. θ gives a genus bound (unproven yet with confidence). We hope (with reason) it says something about ribbon knots.

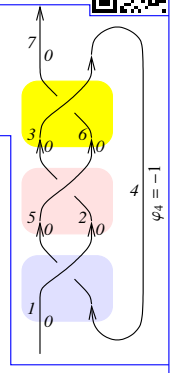
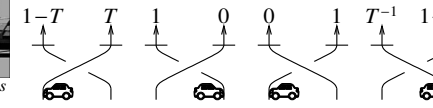
Conventions. T , T_1 , and T_2 are indeterminates and $T_3 := T_1 T_2$.

Preparation. Draw an n -crossing knot K as a diagram D as on the right: all crossings face up, and the edges are marked with a running index $k \in \{1, \dots, 2n+1\}$ and with rotation numbers φ_k .

Model T Traffic Rules. Cars always drive forward. When a car crosses over a sign- s bridge it goes through with (algebraic) probability $T^s \sim 1$, but falls off with probability $1 - T^s \sim 0$. At the very end, cars fall off and disappear. On various edges **traffic counters** are placed. See also [Jo, LTW].



$$p = 1 - T^s$$



Definition. The **traffic function** $G = (g_{\alpha\beta})$ (also, the **Green function** or the **two-point function**) is the reading of a traffic counter at β , if car traffic is injected at α (if $\alpha = \beta$, the counter is *after* the injection point). There are also model- T_v traffic functions $G_v = (g_{v\alpha\beta})$ for $v = 1, 2, 3$.

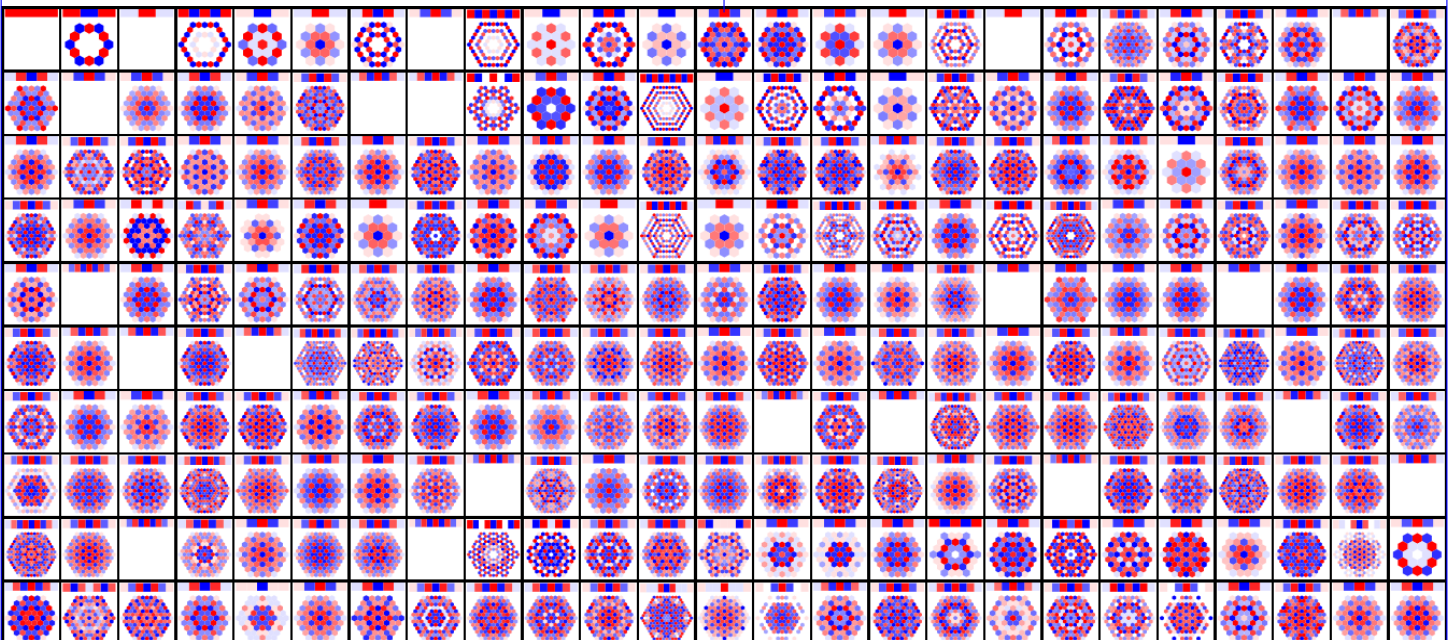
Example.

$$\sum_{p \geq 0} (1-T)^p = T^{-1}$$

$$G = \begin{pmatrix} 1 & T^{-1} & 1 \\ 0 & T^{-1} & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Don’t Look.

$$R_{11}(c) = s \left[1/2 - g_{3ii} + T_2^s g_{1ii} g_{2ji} - T_2^s g_{3jj} g_{2ji} - (T_2^s - 1) g_{3ii} g_{2ji} \right. \\ \left. + (T_3^s - 1) g_{2ji} g_{3ji} - g_{1ii} g_{2jj} + 2 g_{3ii} g_{2jj} + g_{1ii} g_{3jj} - g_{2ii} g_{3jj} \right] \\ + \frac{s}{T_2^s - 1} \left[(T_1^s - 1) T_2^s (g_{3jj} g_{1ji} - g_{2ji} g_{1ji} + T_2^s g_{1ji} g_{2ji}) \right. \\ \left. + (T_3^s - 1) (g_{3ji} - T_2^s g_{1ii} g_{3ji} + g_{2ij} g_{3ji} + (T_2^s - 2) g_{2jj} g_{3ji}) \right. \\ \left. - (T_1^s - 1) (T_2^s + 1) (T_3^s - 1) g_{1ji} g_{3ji} \right] \\ R_{12}(c_0, c_1) = \frac{s_1 (T_1^{s_0} - 1) (T_3^{s_1} - 1) g_{1ji_0} g_{3j_0 i_1}}{T_2^{s_1} - 1} (T_2^{s_0} g_{2i_1 i_0} + g_{2j_1 j_0} - T_2^{s_0} g_{2j_1 i_0} - g_{2i_1 j_0}) \\ \Gamma_1(\varphi, k) = \varphi(-1/2 + g_{3kk})$$



2025/03/10@16:38