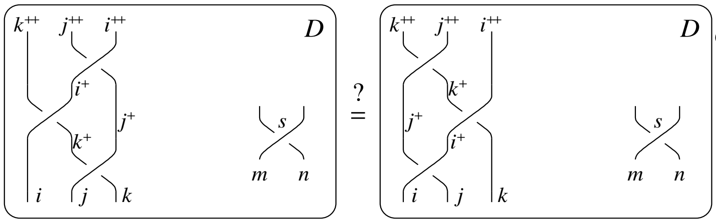
Corollary 2. Proving invariance is easy:



Invariance under R3

This is Theta.nb of http://drorbn.net/to24/ap.

```
Once[<< KnotTheory`; << Rot.m; << PolyPlot.m];
```

```
T3 = T1 T2;
```

```
CF[ℰ_] :=
  Module[{vs = Union@Cases[ℰ, g__, ∞], ps, c},
   Total[CoefficientRules[Expand[ℰ], vs] /.
     (ps_ → c_) :→ Factor[c] (Times @@ vs^ps)] ];
```

```
R11[{s_, i_, j_}] =
  CF[
   s (1/2 - g3ii + T2^s g1ii g2ji - g1ii g2jj -
      (T2^s - 1) g2ji g3ii + 2 g2jj g3ii - (1 - T3^s) g2ji g3ji -
      g2ii g3jj - T2^s g2ji g3jj + g1ii g3jj +
      ((T1^s - 1) g1ji (T2^(2s) g2ji - T2^s g2jj + T2^s g3jj) +
        (T3^s - 1) g3ji
         (1 - T2^s g1ii - (T1^s - 1) (T2^s + 1) g1ji +
           (T2^s - 2) g2jj + g2ij)) / (T2^s - 1))];
```

```
R12[{s0_, i0_, j0_}, {s1_, i1_, j1_}] :=
 CF[s1 (T1^s0 - 1) (T2^s1 - 1)^-1 (T3^s1 - 1) g1,j1,i0 g3,j0,i1
   ((T2^s0 g2,i1,i0 - g2,i1,j0) - (T2^s0 g2,j1,i0 - g2,j1,j0))]
```

```
Γ1[φ_, k_] = -φ/2 + φ g3kk;
```

```
δi_,j_ := If[i === j, 1, 0];
gRs_,i_,j_ := {
   gν_jβ_ :→ gνj+β + δjβ,
   gν_iβ_ :→ Tν^s gνi+β + (1 - Tν^s) gνj+β + δiβ,
   gν_α_i+ :→ Tν^s gναi + δαi+,
   gν_α_j+ :→ gναj + (1 - Tν^s) gναi + δαj+
  }
```

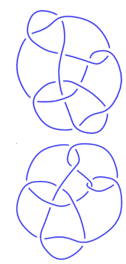
```
DSum[Cs___] := Sum[R11[c], {c, {Cs}}] +
   Sum[R12[c0, c1], {c0, {Cs}}, {c1, {Cs}}]
lhs = DSum[{1, j, k}, {1, i, k+}, {1, i+, j+},
     {s, m, n}] //. gR1,j,k ⋃ gR1,i,k+ ⋃ gR1,i+,j+;
rhs = DSum[{1, i, j}, {1, i+, k}, {1, j+, k+},
     {s, m, n}] //. gR1,i,j ⋃ gR1,i+,k ⋃ gR1,j+,k+;
Simplify[lhs == rhs]
```

```
True
```

The Main Program

```
Θ[K_] := Module[{Cs, φ, n, A, Δ, G, ev, θ},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :→
    (A[[{i, j}, {i + 1, j + 1}]] += ( -T^s  T^s - 1
                                       0     -1   ))];
  Δ = T^((-Total[φ] - Total[Cs[[All,1]]])/2) Det[A];
  G = Inverse[A];
  ev[ℰ_] :=
   Factor[ℰ /. gν_,α_,β_ :→ (G[[α, β]] /. T → Tν)];
  θ = ev[Σ_{k1=1}^n Σ_{k2=1}^n R12[Cs[[k1]], Cs[[k2]]]];
  θ += ev[Σ_{k=1}^n R11[Cs[[k]]]];
  θ += ev[Σ_{k=1}^{2n} Γ1[φ[[k]], k]];
  Factor@
   {Δ, (Δ /. T → T1) (Δ /. T → T2) (Δ /. T → T3) θ}];
```

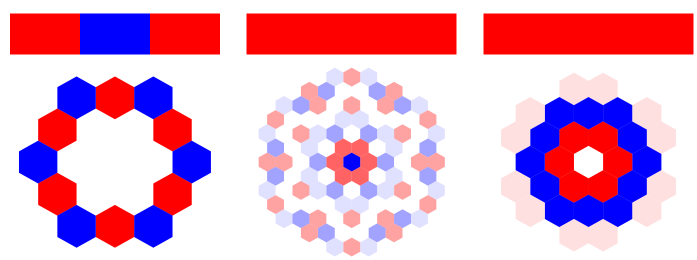
The Trefoil, Conway, and Kinoshita-Terasaka



```
Θ[Knot[3, 1]] // Expand
```

$$\left\{-1+\frac{1}{T}+T,\ -\frac{1}{T_1^2}-T_1^2-\frac{1}{T_2^2}-\frac{1}{T_1^2 T_2^2}+\frac{1}{T_1 T_2^2}+\frac{1}{T_1^2 T_2}+\frac{T_1}{T_2}+\frac{T_2}{T_1}+T_1^2 T_2-T_2^2+T_1 T_2^2-T_1^2 T_2^2\right\}$$

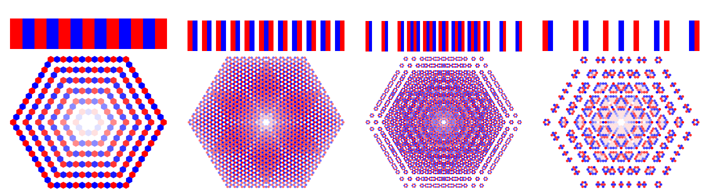
```
GraphicsRow[PolyPlot[Θ[Knot[#]]] & /@
  {"3_1", "K11n34", "K11n42"}]
```

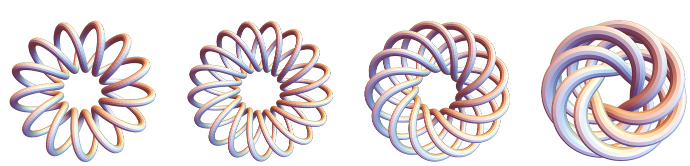


(Note that the genus of the Conway knot appears to be bigger than the genus of Kinoshita-Terasaka)

Some Torus Knots

```
TKs = {{13, 2}, {17, 3}, {13, 5}, {7, 6}};
GraphicsRow[PolyPlot[Θ[TorusKnot @@ #]] & /@ TKs]
GraphicsRow[TubePlot[TorusKnot @@ #] & /@ TKs]
```





Video and more at http://www.math.toronto.edu/~drorbn/Talks/Toronto-241030.