Implementation (see IType.nb of ωεβ/ap).

```
☺ Once[<< KnotTheory`; << Rot.m];
```

```
C:\drorbn\AcademicPensieve\Projects\KnotTheory\KnotTheory
Loading KnotTheory` version
    of February 2, 2020, 10:53:45.2097.
  Read more at http://katlas.org/wiki/KnotTheory.
Loading Rot.m from
    http://drorbn.net/AP/Talks/Geneva-2408
    to compute rotation numbers.
```

```
☺ CF[ω_. ℰ_𝔼] := CF[ω] × CF /@ ℰ;
  CF[ℰ_List] := CF /@ ℰ;
  CF[ℰ_] := Module[{vs, ps, c},
     vs = Cases[ℰ, (x | p | ξ | π | g)__, ∞] ⋃ {ϵ};
     Total[CoefficientRules[Expand[ℰ], vs] /.
       (ps_ → c_) :→ Factor[c] (Times @@ vs^ps)] ];
```

Integration using Picard iteration. The core is in yellow and hacks are in pink.

```
☺ 𝔼 /: 𝔼[A_] × 𝔼[B_] := 𝔼[A + B];
☺ $π = Identity; (* The Wisdom Projection *)
☺ Unprotect[Integrate];
  ∫ω_. 𝔼[L_] ⅆ(vs_List) :=
    Module[{n, L0, Q, Δ, G, Z0, Z, λ, DZ, DDZ, FZ,
      a, b},
     n = Length@vs; L0 = L /. ϵ → 0;
     Q = Table[(-∂_{vs[[a]],vs[[b]]} L0) /. Thread[vs → 0] /.
        (p | x)__ → 0, {a, n}, {b, n}];
     If[(Δ = Det[Q]) == 0, Return@"Degenerate Q!"];
     Z = Z0 = CF@$π[L + vs.Q.vs / 2]; G = Inverse[Q];
     FixedPoint[(DZ = Table[∂_v Z, {v, vs}];
        DDZ = Table[∂_u DZ, {u, vs}];
        FZ = Sum[G[[a, b]] (DDZ[[a, b]] + DZ[[a]] × DZ[[b]]),
          {a, n}, {b, n}] / 2;
        Z = CF[Z0 + ∫_0^λ $π[FZ] ⅆλ]) &, Z];
     PowerExpand@Factor[ω Δ^(-1/2)] ×
      𝔼[CF[Z /. λ → 1 /. Thread[vs → 0]]]];
  Protect[Integrate];
```

☺ $\int \mathbb{E}\left[-\mu x^2/2 + i\,\xi\, x\right] d\{x\}$

$$\frac{\mathbb{E}\left[-\frac{\xi^2}{2\mu}\right]}{\sqrt{\mu}}$$

☺ $\text{FofG} = \int \mathbb{E}\left[-\mu\,(x-a)^2/2 + i\,\xi\, x\right] d\{x\}$

$$\frac{\mathbb{E}\left[\frac{i\,(2\,a\,\mu + i\,\xi)\,\xi}{2\,\mu}\right]}{\sqrt{\mu}}$$

Joseph Fourier

☺ $\int \text{FofG}\, \mathbb{E}[-i\,\xi\, x]\, d\{\xi\}$

$$\mathbb{E}\left[-\frac{1}{2}(a-x)^2\,\mu\right]$$

So we've tested and nearly proven the Fourier inversion formula!

☺ $L = -\frac{1}{2}\{x_1, x_2\}.\begin{pmatrix} a & b \\ b & c \end{pmatrix}.\{x_1, x_2\} + \{\xi_1, \xi_2\}.\{x_1, x_2\};$

$Z12 = \int \mathbb{E}[L]\, d\{x_1, x_2\}$

$$\frac{\mathbb{E}\left[\frac{c\,\xi_1^2}{2\,(-b^2+a\,c)} + \frac{b\,\xi_1\,\xi_2}{b^2-a\,c} + \frac{a\,\xi_2^2}{2\,(-b^2+a\,c)}\right]}{\sqrt{-b^2+a\,c}}$$

☺ $\left\{Z1 = \int \mathbb{E}[L]\, d\{x_1\},\ Z12 == \int Z1\, d\{x_2\}\right\}$

$$\left\{\frac{\mathbb{E}\left[-\frac{(-b^2+a\,c)\,x_2^2}{2\,a} - \frac{b\,x_2\,\xi_1}{a} + \frac{\xi_1^2}{2\,a} + x_2\,\xi_2\right]}{\sqrt{a}},\ \text{True}\right\}$$

Guido Fubini

☺ $\$\pi = \text{Normal}\left[\#\ +\ O[\epsilon]^{13}\right]\ \&;\ \int \mathbb{E}\left[-\phi^2/2 + \epsilon\,\phi^3/6\right] d\{\phi\}$

$$\mathbb{E}\left[\frac{5\,\epsilon^2}{24} + \frac{5\,\epsilon^4}{16} + \frac{1105\,\epsilon^6}{1152} + \frac{565\,\epsilon^8}{128} + \frac{82\,825\,\epsilon^{10}}{3072} + \frac{19\,675\,\epsilon^{12}}{96}\right]$$

From https://oeis.org/A226260:

THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES ®

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Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

A226260 Numerators of mass formula for connected vacuum graphs on 2n nodes for a phi^3 field theory.

1, 5, 5, 1105, 565, 82825, 19675, 1282031525, 80727925, 1683480621875, 13209845125, 2239646759308375, 19739117098375, 632079170908309375, 32468078556378125, 3836267676884504575187 5, 2813657784050329731 25, 2824650747089425586152484375, 7766321570341167127343 75 (list; graph; refs; listen; history; text; internal format)

The Right-Handed Trefoil.

```
☺ K = Mirror@Knot[3, 1]; Features[K]
Features[7, C4[-1] X1,5[1] X3,7[1] X6,2[1]]
☺ ℒ[X_{i_,j_}[s_]] := T^(s/2) 𝔼[
     x_i (p_{i+1} - p_i) + x_j (p_{j+1} - p_j) +
      (T^s - 1) x_i (p_{i+1} - p_{j+1}) +
      (ϵ s / 2) ×
       (x_i (p_i - p_j) ((T^s - 1) x_i p_j + 2 (1 - x_j p_j)) - 1)]
  ℒ[C_i_[φ_]] := T^(φ/2) 𝔼[x_i (p_{i+1} - p_i) + ϵ φ (1/2 - x_i p_i)]
  ℒ[K_] := CF[ℒ /@ Features[K][[2]]]
  vs[K_] :=
   Join @@ Table[{p_i, x_i}, {i, Features[K][[1]]}]
```