☺ $\{\mathtt{vs}[K],\ \mathcal{L}[K]\}$

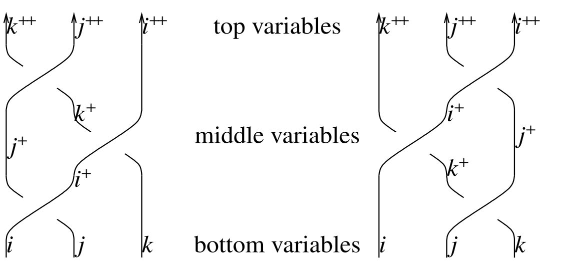
$$\Big\{\{p_1, x_1, p_2, x_2, p_3, x_3, p_4, x_4, p_5, x_5, p_6, x_6, p_7, x_7\},\ T\,\mathbb{E}\Big[-2\epsilon - p_1 x_1 + \epsilon p_1 x_1 + T p_2 x_1 - \epsilon p_5 x_1 + (1-T) p_6 x_1 + \frac{1}{2}(-1+T)\epsilon p_1 p_5 x_1^2 + \frac{1}{2}(1-T)\epsilon p_5^2 x_1^2 - p_2 x_2 + p_3 x_2 - p_3 x_3 + \epsilon p_3 x_3 + T p_4 x_3 - \epsilon p_7 x_3 + (1-T) p_8 x_3 + \frac{1}{2}(-1+T)\epsilon p_3 p_7 x_3^2 + \frac{1}{2}(1-T)\epsilon p_7^2 x_3^2 - p_4 x_4 + \epsilon p_4 x_4 + p_5 x_4 - p_5 x_5 + p_6 x_5 - \epsilon p_1 p_5 x_1 x_5 + \epsilon p_5^2 x_1 x_5 - \epsilon p_2 x_6 + (1-T) p_3 x_6 - p_6 x_6 + \epsilon p_6 x_6 + T p_7 x_6 + \epsilon p_2^2 x_2 x_6 - \epsilon p_2 p_6 x_2 x_6 + \frac{1}{2}(1-T)\epsilon p_2^2 x_6^2 + \frac{1}{2}(-1+T)\epsilon p_2 p_6 x_6^2 - p_7 x_7 + p_8 x_7 - \epsilon p_3 p_7 x_3 x_7 + \epsilon p_7^2 x_3 x_7\Big]\Big\}$$

☺ $\$\pi = \mathtt{Normal}\left[\# + O[\epsilon]^2\right]\ \&;\ \int \mathcal{L}[K]\, d\, \mathtt{vs}[K]$

$$-\frac{i\, T\, \mathbb{E}\left[-\frac{(-1+T)^2 (1+T^2)\epsilon}{(1-T+T^2)^2}\right]}{1-T+T^2}$$

A faster program to compute ρ_1, and more stories about it, are at [BV2].

Invariance Under Reidemeister 3.



Reidemeister

☺ $\mathtt{lhs} = \int (\mathcal{L} /@ (X_{i,j}[1]\, X_{i+1,k}[1]\, X_{j+1,k+1}[1]))\ d\{p_{i+1}, p_{j+1}, p_{k+1}, x_{i+1}, x_{j+1}, x_{k+1}\};$

$\mathtt{rhs} = \int (\mathcal{L} /@ (X_{j,k}[1]\, X_{i,k+1}[1]\, X_{i+1,j+1}[1]))\ d\{x_{i+1}, p_{i+1}, p_{j+1}, p_{k+1}, x_{j+1}, x_{k+1}\};$

$\mathtt{lhs} === \mathtt{rhs}$

False

Invariance Under Reidemeister 3, Take 2.

☺ $\mathtt{lhs} = \int (\mathcal{L} /@ (X_{i,j}[1]\, X_{i+1,k}[1]\, X_{j+1,k+1}[1]))\ d\{x_i, x_j, x_k, p_{i+1}, p_{j+1}, p_{k+1}, x_{i+1}, x_{j+1}, x_{k+1}\};$

$\mathtt{rhs} = \int (\mathcal{L} /@ (X_{j,k}[1]\, X_{i,k+1}[1]\, X_{i+1,j+1}[1]))\ d\{x_i, x_j, x_k, x_{i+1}, p_{i+1}, p_{j+1}, p_{k+1}, x_{j+1}, x_{k+1}\};$

$\mathtt{lhs} === \mathtt{rhs}$

True

☺ lhs

Degenerate Q!

Invariance Under Reidemeister 3, Take 3.

☺ $\mathtt{lhs} = \int (\mathbb{E}[i\pi_i p_i + i\pi_j p_j + i\pi_k p_k] \times \mathcal{L} /@ (X_{i,j}[1]\, X_{i+1,k}[1]\, X_{j+1,k+1}[1]))\ d\{p_i, p_j, p_k, x_i, x_j, x_k, p_{i+1}, p_{j+1}, p_{k+1}, x_{i+1}, x_{j+1}, x_{k+1}\};$

$\mathtt{rhs} = \int (\mathbb{E}[i\pi_i p_i + i\pi_j p_j + i\pi_k p_k] \times \mathcal{L} /@ (X_{j,k}[1]\, X_{i,k+1}[1]\, X_{i+1,j+1}[1]))\ d\{p_i, p_j, p_k, x_i, x_j, x_k, p_{i+1}, p_{j+1}, p_{k+1}, x_{i+1}, x_{j+1}, x_{k+1}\};$

$\mathtt{lhs} == \mathtt{rhs}$

True

☺ lhs

$$T^{3/2}\,\mathbb{E}\Big[-\frac{3\epsilon}{2} + i T^2 p_{2+i}\pi_i - i(-1+T)T p_{2+j}\pi_i + iT^2\epsilon p_{2+j}\pi_i - i(-1+T)p_{2+k}\pi_i + iT\epsilon p_{2+k}\pi_i - \frac{1}{2}(-1+T)T^3\epsilon p_{2+i}p_{2+j}\pi_i^2 + \frac{1}{2}(-1+T)T^3\epsilon p_{2+j}^2\pi_i^2 - \frac{1}{2}(-1+T)T^2\epsilon p_{2+i}p_{2+k}\pi_i^2 + \frac{1}{2}(-1+T)^2 T\epsilon p_{2+j}p_{2+k}\pi_i^2 + \frac{1}{2}(-1+T)T\epsilon p_{2+k}^2\pi_i^2 + iTp_{2+j}\pi_j - iT\epsilon p_{2+j}\pi_j - i(-1+T)p_{2+k}\pi_j + i(-1+2T)\epsilon p_{2+k}\pi_j + T^3\epsilon p_{2+i}p_{2+j}\pi_i\pi_j - T^3\epsilon p_{2+j}^2\pi_i\pi_j - (-1+T)T^2\epsilon p_{2+i}p_{2+k}\pi_i\pi_j + (-1+T)^2 T\epsilon p_{2+j}p_{2+k}\pi_i\pi_j + (-1+T)T\epsilon p_{2+k}^2\pi_i\pi_j - \frac{1}{2}(-1+T)T\epsilon p_{2+j}p_{2+k}\pi_j^2 + \frac{1}{2}(-1+T)T\epsilon p_{2+k}^2\pi_j^2 + i p_{2+k}\pi_k - 2i\epsilon p_{2+k}\pi_k + T^2\epsilon p_{2+i}p_{2+k}\pi_i\pi_k - (-1+T)T\epsilon p_{2+j}p_{2+k}\pi_i\pi_k - T\epsilon p_{2+k}^2\pi_i\pi_k + T\epsilon p_{2+j}p_{2+k}\pi_j\pi_k - T\epsilon p_{2+k}^2\pi_j\pi_k\Big]$$

Invariance under the other Reidemeister moves is proven in a similar way. See IType.nb at ωεβ/ap.

There's more! To get sl_2 invariants mod ϵ^3, add the following to $L(X_{ij}^+)$, $L(X_{ij}^-)$, and $L(C_i^\varphi)$, respectively (and see More.nb at ωεβ/ap for the verifications):

☺ $\epsilon^2\, r_2[1, i, j]$

$$\frac{1}{12}\epsilon^2\Big(-6p_i x_i + 6p_j x_i - 3(-1+3T)p_i p_j x_i^2 + 3(-1+3T)p_j^2 x_i^2 + 4(-1+T)p_i^2 p_j x_i^3 - 2(-1+T)(5+T)p_i p_j^2 x_i^3 + 2(-1+T)(3+T)p_j^3 x_i^3 + 18 p_i p_j x_i x_j - 18 p_j^2 x_i x_j - 6 p_i^2 p_j x_i^2 x_j + 6(2+T)p_i p_j^2 x_i^2 x_j - 6(1+T)p_j^3 x_i^2 x_j - 6 p_i p_j^2 x_i x_j^2 + 6 p_j^3 x_i x_j^2\Big)$$

☺ $\epsilon^2\, r_2[-1, i, j]$

$$\frac{1}{12T^2}\epsilon^2\Big(-6T^2 p_i x_i + 6T^2 p_j x_i + 3(-3+T)T p_i p_j x_i^2 - 3(-3+T)T p_j^2 x_i^2 - 4(-1+T)T p_i^2 p_j x_i^3 + 2(-1+T)(1+5T)p_i p_j^2 x_i^3 - 2(-1+T)(1+3T)p_j^3 x_i^3 + 18T^2 p_i p_j x_i x_j - 18T^2 p_j^2 x_i x_j - 6T^2 p_i^2 p_j x_i^2 x_j + 6T(1+2T)p_i p_j^2 x_i^2 x_j - 6T(1+T)p_j^3 x_i^2 x_j - 6T^2 p_i p_j^2 x_i x_j^2 + 6T^2 p_j^3 x_i x_j^2\Big)$$

☺ $\epsilon^2\, \gamma_2[\varphi, i]$

$$-\frac{1}{2}\epsilon^2\varphi^2 p_i x_i$$

Even more! • The sl_2 formulas mod ϵ^4 are in the last page of the handout of [BN3].

• Using [GPV] we can show that every finite type invariant is I-Type.

• Probably, ⟨Reshetikhin-Turaev⟩ ⊂ ⟨I-Type⟩ efficiently.

• Possibly, ⟨Rozansky Polynomials⟩ ⊂ ⟨I-Type⟩ efficiently.

• Knot signatures are I-Type, at least mod 8.

• We already have some work on sl_3, and it leads to the strongest genuinely-computable knot invariant presently known.

Video and more at http://www.math.toronto.edu/~drorbn/Talks/Beijing-2407 and http://www.math.toronto.edu/~drorbn/Talks/Geneva-2408.