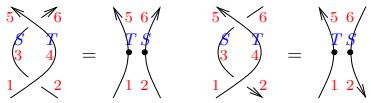
Reidemeister 2



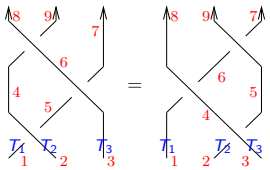
```
𝒜@{X_{2,4,3,1}[S, T], X̄_{3,4,6,5}} ≡ 𝒜@{P_{1,5}[T], P_{2,6}[S]}
```

```
True
```

```
𝒜@{X̄_{3,1,2,4}[S, T], X_{6,5,3,4}} ≡ 𝒜@{P_{1,5}[T], P_{2,6}[S]}
```

```
True
```

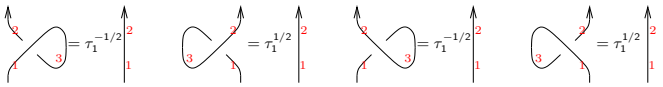
Reidemeister 3



```
𝒜@{X_{2,5,4,1}[T_2, T_1], X_{3,7,6,5}[T_3, T_1], X_{6,9,8,4}} ≡
 𝒜@{X_{3,5,4,2}[T_3, T_2], X_{4,6,8,1}[T_3, T_1], X_{5,7,9,6}}
```

```
True
```

Reidemeister 1

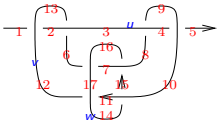


```
{𝒜@{X_{3,3,2,1}} ≡ τ_1^{-1/2} 𝒜@{P_{1,2}}, 𝒜@{X_{1,2,3,3}} ≡ τ_1^{1/2} 𝒜@{P_{1,2}},
 𝒜@{X̄_{1,3,3,2}} ≡ τ_1^{-1/2} 𝒜@{P_{1,2}}, 𝒜@{X̄_{3,1,2,3}} ≡ τ_1^{1/2} 𝒜@{P_{1,2}}}
```

```
{True, True, True, True}
```

(So we have an invariant, up to rotation numbers).

The Relation with the Multivariable Alexander Polynomial



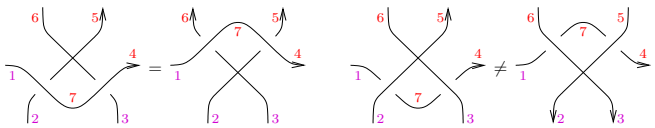
```
MVA = u^{-1/2} v^{-1/2} w^{-1/2} (u - 1) (v - 1) (w - 1);
A = {X̄_{1,12,2,13}[u, v], X̄_{13,2,6,3}, X_{8,4,9,3}, X_{4,10,5,9}, X_{6,17,7,16}[v, w],
      X_{15,8,16,7}, X̄_{14,10,15,11}, X̄_{11,17,12,14}} // 𝒜 // Last // Factor
```

$$\frac{(-1+u)^2\,(-1+v)\,(-1+w)\,(\text{Wedge}[\,]-x_5\wedge\xi_1)}{u\,v}$$

```
A == u^{-1/2} (u - 1) u^0 v^{-1/2} w^{1/2} MVA (Wedge[∧] - x_5 ∧ ξ_1)
```

```
True
```

Overcrossings Commute but Undercrossings don't



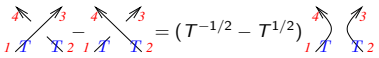
```
𝒜@{X_{2,7,5,1}, X_{3,4,6,7}} ≡ 𝒜@{X_{3,7,6,1}, X_{2,4,5,7}}
```

```
True
```

```
𝒜@{X̄_{1,2,7,5}, X̄_{7,3,4,6}} ≡ 𝒜@{X̄_{1,3,7,6}, X̄_{7,2,4,5}}
```

```
False
```

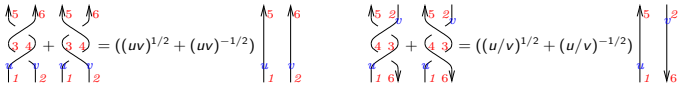
The Conway Relation (see [Co])



```
𝒜@{X_{2,3,4,1}[T, T]} - 𝒜@{X̄_{1,2,3,4}[T, T]} ≡ (T^{-1/2} - T^{1/2}) 𝒜@{P_{1,4}[T], P_{2,3}[T]}
```

```
True
```

Conway's Second Set of Identities (see [Co])



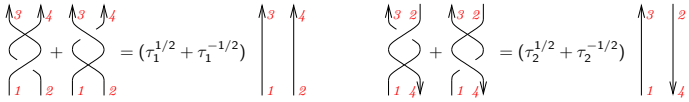
```
𝒜@{X_{2,4,3,1}[v, u], X_{4,6,5,3}} + 𝒜@{X̄_{1,2,4,3}[u, v], X̄_{3,4,6,5}} ≡
 (u^{1/2} v^{1/2} + u^{-1/2} v^{-1/2}) 𝒜@{P_{1,5}[u], P_{2,6}[v]}
```

```
True
```

```
𝒜@{X̄_{4,1,6,3}[v, u], X̄_{3,2,5,4}} + 𝒜@{X_{1,6,3,4}[u, v], X_{2,5,4,3}} ≡
 (u^{1/2} v^{-1/2} + u^{-1/2} v^{1/2}) 𝒜@{P_{1,5}[u], P_{2,6}[v]}
```

```
True
```

Virtual versions (Archibald, [Ar])



```
𝒜@{X_{2,3,4,1}} + 𝒜@{X̄_{2,1,4,3}} ≡ (τ_1^{1/2} + τ_1^{-1/2}) 𝒜@{P_{1,3}, P_{2,4}}
```

```
True
```

```
𝒜@{X̄_{1,2,3,4}} + 𝒜@{X_{1,4,3,2}} ≡ (τ_2^{1/2} + τ_2^{-1/2}) 𝒜@{P_{1,3}, P_{2,4}}
```

```
True
```

Video and more at http://www.math.toronto.edu/~drorbn/Talks/MoscowByWeb-2104/