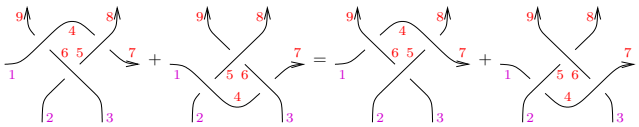
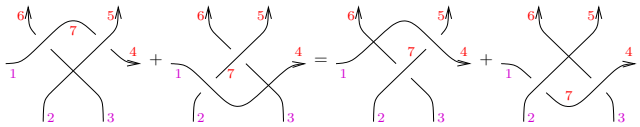
Conway's Third Identity (see [Co])



$\mathcal{A}@\{X_{6,4,9,1}, \overline{X}_{4,5,7,8}, \overline{X}_{2,3,5,6}\} + \mathcal{A}@\{X_{2,4,5,1}, \overline{X}_{4,3,7,6}, X_{6,8,9,5}\} \equiv \mathcal{A}@\{\overline{X}_{1,6,4,9}, X_{5,7,8,4}, X_{3,5,6,2}\} + \mathcal{A}@\{\overline{X}_{1,2,4,5}, X_{3,7,6,4}, \overline{X}_{5,6,8,9}\}$

True

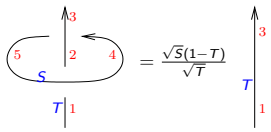
Virtual version (Archibald, [Ar])



$\mathcal{A}@\{X_{3,7,6,1}, \overline{X}_{7,2,4,5}\} + \mathcal{A}@\{X_{2,4,7,1}, X_{3,5,6,7}\} \equiv \mathcal{A}@\{X_{3,7,6,2}, X_{7,4,5,1}\} + \mathcal{A}@\{\overline{X}_{1,2,7,5}, X_{3,4,6,7}\}$

True

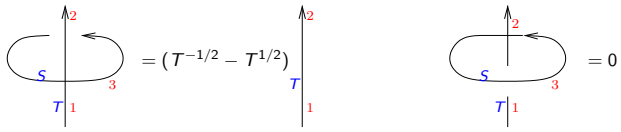
Jun Murakami's Fifth Axiom (see [Mu])



$\mathcal{A}@\{X_{1,4,2,5}[T, S], X_{4,3,5,2}\} \equiv \frac{\sqrt{S}\ (1 - T)}{\sqrt{T}}\ \mathcal{A}@\{P_{1,3}[T]\}$

True

Virtual versions (Archibald, [Ar])



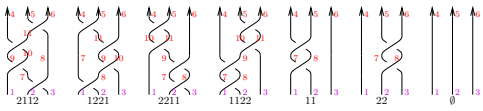
$\mathcal{A}@\{X_{3,2,3,1}[S, T]\} \equiv \left(T^{-1/2} - T^{1/2}\right)\ \mathcal{A}@\{P_{1,2}[T]\}$

True

$\mathcal{A}@\{X_{1,3,2,3}\}$

$\mathcal{A}[\{1\}, \{2\}, \langle | \xi_1 \to \tau_1, x_2 \to \tau_1 | \rangle, 0]$

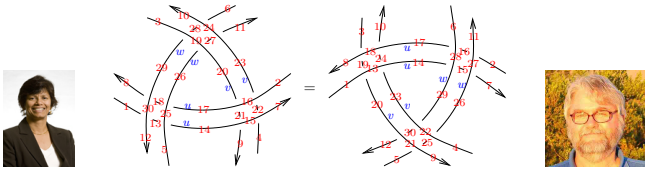
Jun Murakami's Third Axiom (see [Mu])



$\mathcal{A}_{2112} = \mathcal{A}@\{X_{3,8,7,2}, X_{7,10,9,1}, X_{10,11,4,9}, X_{8,6,5,11}\};$
$\mathcal{A}_{1221} = \mathcal{A}@\{X_{2,8,7,1}, X_{3,10,9,8}, X_{10,6,11,9}, X_{11,5,4,7}\};$
$\mathcal{A}_{2211} = \mathcal{A}@\{X_{3,8,7,2}, X_{8,6,9,7}, X_{9,11,10,1}, X_{11,5,4,10}\};$
$\mathcal{A}_{1122} = \mathcal{A}@\{X_{2,8,7,1}, X_{8,9,4,7}, X_{3,11,10,9}, X_{11,6,5,10}\};$
$\mathcal{A}_{11} = \mathcal{A}@\{X_{2,8,7,1}, X_{8,5,4,7}, P_{3,6}\};\ \ \mathcal{A}_{22} = \mathcal{A}@\{X_{3,8,7,2}, X_{8,6,5,7}, P_{1,4}\};$
$\mathcal{A}_{\emptyset} = \mathcal{A}@\{P_{1,4}, P_{2,5}, P_{3,6}\};$
$g_+[z_] := z^{1/2} + z^{-1/2};\ g_-[z_] := z^{1/2} - z^{-1/2};$
$g_+[\tau_1]\ g_-[\tau_2]\ \mathcal{A}_{2112} - g_-[\tau_2]\ g_+[\tau_3]\ \mathcal{A}_{1221} - g_-[\tau_3 / \tau_1]\ (\mathcal{A}_{2211} + \mathcal{A}_{1122}) + g_-[\tau_2\ \tau_3 / \tau_1]\ g_+[\tau_3]\ \mathcal{A}_{11} - g_+[\tau_1]\ g_-[\tau_1\ \tau_2 / \tau_3]\ \mathcal{A}_{22} \equiv g_-\left[\tau_3^2 / \tau_1^2\right]\ \mathcal{A}_{\emptyset}$

True

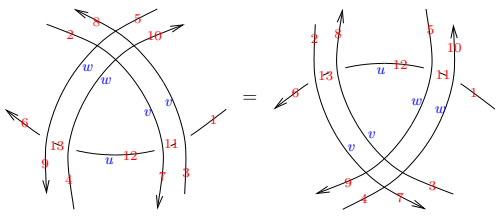
The Naik-Stanford Double Delta Move (see [NS])



$\texttt{Timing}\left[\mathcal{A}@\{X_{6,10,28,24}[w, v], \overline{X}_{28,3,29,19}[w, v], X_{26,20,27,19}[w, v], \overline{X}_{27,23,11,24}[w, v], X_{1,12,13,30}[u, w], \overline{X}_{13,5,14,25}[u, w], X_{17,26,18,25}[u, w], \overline{X}_{18,29,8,30}[u, w], X_{4,7,22,15}[v, u], \overline{X}_{22,2,23,16}[v, u], X_{20,17,21,16}[v, u], \overline{X}_{21,14,9,15}[v, u]\} \equiv \mathcal{A}@\{X_{5,9,25,21}[w, v], \overline{X}_{25,4,26,22}[w, v], X_{29,23,30,22}[w, v], \overline{X}_{30,20,12,21}[w, v], X_{2,11,16,27}[u, w], \overline{X}_{16,6,17,28}[u, w], X_{14,29,15,28}[u, w], \overline{X}_{15,26,7,27}[u, w], X_{3,8,19,18}[v, u], \overline{X}_{19,1,20,13}[v, u], X_{23,14,24,13}[v, u], \overline{X}_{24,17,10,18}[v, u]\}\right]$

{190.422, True}

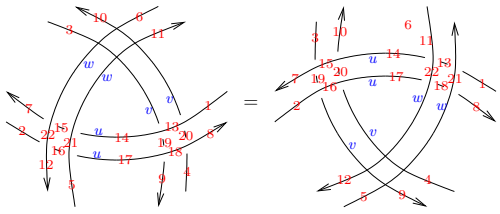
Virtual Version 1 (Archibald, [Ar])



$\mathcal{A}@\{X_{1,8,11,3}[u, v], \overline{X}_{11,2,12,7}[u, v], X_{12,10,13,4}[u, w], \overline{X}_{13,5,6,9}[u, w]\} \equiv \mathcal{A}@\{X_{1,10,11,4}[u, w], \overline{X}_{11,5,12,9}[u, w], X_{12,8,13,3}[u, v], \overline{X}_{13,2,6,7}[u, v]\}$

True

Virtual Version 2 (Archibald, [Ar])



$\mathcal{A}@\{\overline{X}_{20,1,10,13}[v, u], X_{3,14,19,13}[v, u], X_{14,11,15,21}[u, w], \overline{X}_{15,6,7,22}[u, w], X_{2,12,16,22}[u, w], \overline{X}_{16,5,17,21}[u, w], \overline{X}_{19,17,9,18}[v, u], X_{4,8,20,18}[v, u]\} \equiv \mathcal{A}@\{X_{1,11,13,21}[u, w], \overline{X}_{13,6,14,22}[u, w], \overline{X}_{20,14,10,15}[v, u], X_{3,7,19,15}[v, u], \overline{X}_{19,2,9,16}[v, u], X_{4,17,20,16}[v, u], X_{17,12,18,22}[u, w], \overline{X}_{18,5,8,21}[u, w]\}$

True

Video and more at http://www.math.toronto.edu/~drorbn/Talks/MoscowByWeb-2104/