Conversions $\mathcal{A} \leftrightarrow \Gamma$:

```
Γ@A[is_, os_, cs_, w_] := Module[{i, j, ω = Coefficient[w, Wedge[◻]]},
   Γ[is, os, cs, ω, Sum[Cancel[-Coefficient[w, x_j ∧ ξ_i] ξ_i x_j / ω],
     {i, is}, {j, os}]]
  ];
A@Γ[is_, os_, cs_, ω_, λ_] :=
  A[is, os, cs, Expand[ω WExp[Expand[λ] /. ξ_a_ x_b_ :> ξ_a ∧ x_b]]];
```

The conversions are inverses of each other:

```
γ = Γ[{1, 2, 3}, {1, 2, 3}, {x1 → τ1, x2 → τ2, x3 → τ3, ξ1 → τ1, ξ2 → τ2, ξ3 → τ3},
   ω, a11 x1 ξ1 + a12 x2 ξ1 + a13 x3 ξ1 + a21 x1 ξ2 + a22 x2 ξ2 + a23 x3 ξ2 + a31 x1 ξ3 +
    a32 x2 ξ3 + a33 x3 ξ3];
Γ@A@γ == γ
```

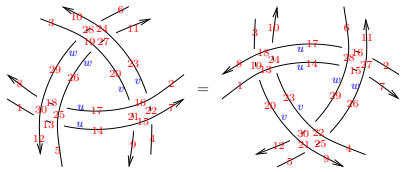
True

The conversions commute with contractions:

```
Γ@c3,3@A@γ ≡ c3,3@γ
```

True

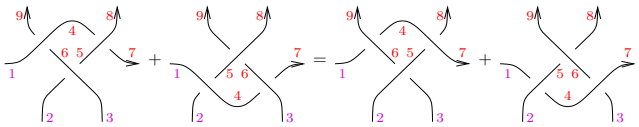
The Naik-Stanford Double Delta Move (again)



```
Timing[Γ@{X6,10,28,24[w, v], X̄28,3,29,19[w, v], X26,20,27,19[w, v], X̄27,23,11,24[w, v],
    X1,12,13,30[u, w], X̄13,5,14,25[u, w], X17,26,18,25[u, w], X̄18,29,8,30[u, w],
    X4,7,22,15[v, u], X̄22,2,23,16[v, u], X20,17,21,16[v, u], X̄21,14,9,15[v, u]} ≡
  Γ@{X5,9,25,21[w, v], X̄25,4,26,22[w, v], X29,23,30,22[w, v], X̄30,20,12,21[w, v],
    X2,11,16,27[u, w], X̄16,6,17,28[u, w], X14,29,15,28[u, w], X̄15,26,7,27[u, w],
    X3,8,19,18[v, u], X̄19,1,20,13[v, u], X23,14,24,13[v, u], X̄24,17,10,18[v, u]}]
```

{0.703125, True}

Conway's Third Identity



Sorry, Γ has nothing to say about that...'

What I still don't understand.

- What becomes of $c_{x,\xi}e^{\lambda}$ if we have to divide by 0 in order to write it again as an exponentiated quadratic? Does it still live within a very small subset of $\Lambda(\mathcal{X} \sqcup X)$?
- How do cablings and strand reversals fit within $\mathcal{A}$?
- Are there "classicality conditions" satisfied by the invariants of classical tangles (as opposed to virtual ones)?

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Thank You!

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