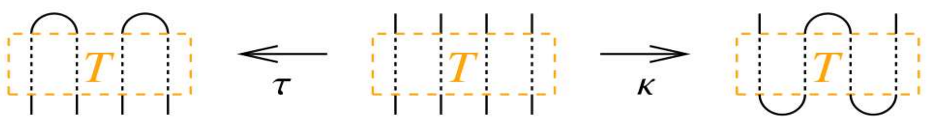
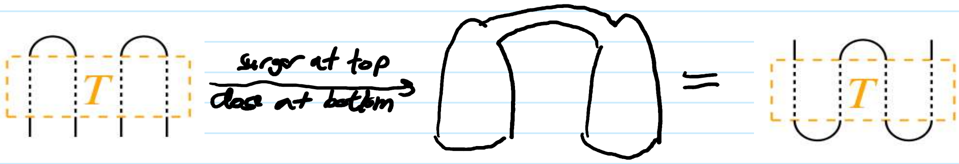
Proof of the Tangle Characterization of Ribbon Knots



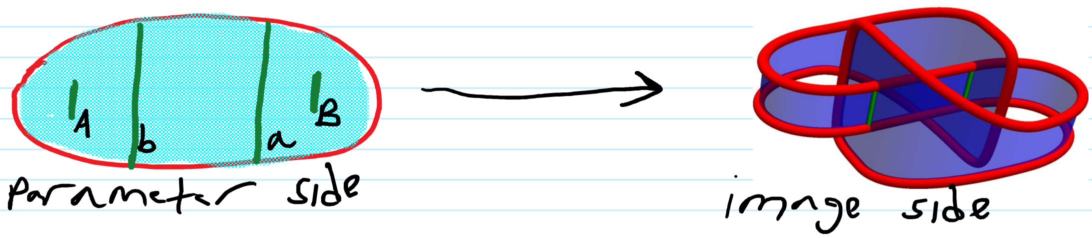
(tangle has 2n strands, here n=2)

Theorem. A knot K is ribbon iff there exists a tangle T whose τ closure is the untangle and whose κ closure is K.

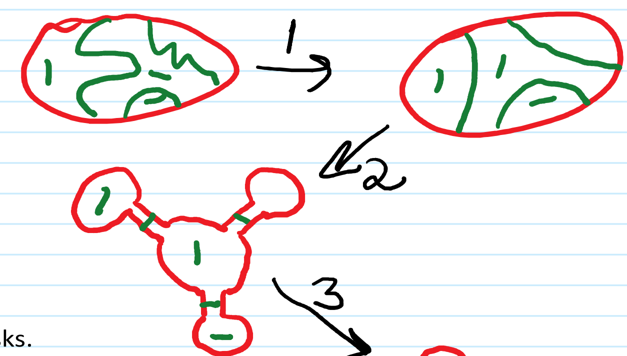
Proof. The backward $\Leftarrow$ implication is easy:



For the forward implication, follow the following 5 steps:

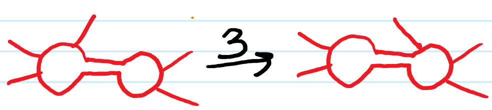


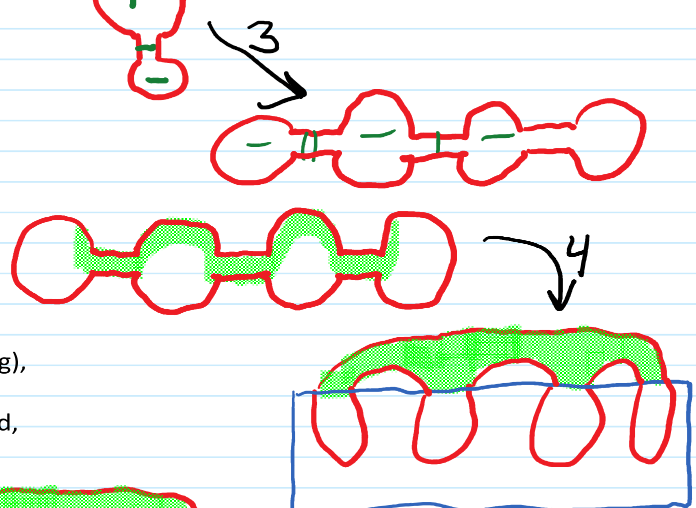
Step I: In-situ cosmetics.
At end: D is a tree of chord-and-arc polygons.



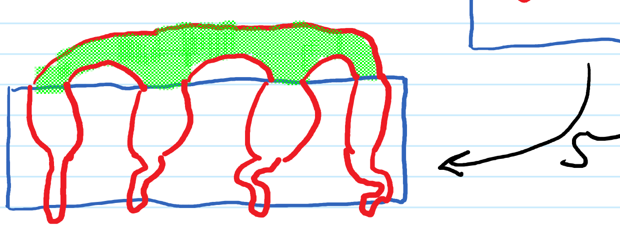
Step 2: Near-situ cosmetics.
At end: D is tree-band-sum of n unknotted disks.

Step 3: Slides.
At end: D is a linear-band-sum of n unknotted disks.





Step 4: Exposure!
The green domain is contractible - so it can be shrank, moved at will (with the blue membrane following along), and expanded back again.
At end: D has (n-1) exposed bridges which when turned, make D a union of n unknotted disks.



Step 5: Pulling bottom handles avoiding the obstacles.
At end: Theorem is proven.