The Real Thing. In the algebra QU_ϵ, over $\mathbb{Q}[\![\hbar]\!]$ using the *yaxt* order, $T = \mathbb{e}^{\hbar t}$, $\bar{T} = T^{-1}$, $\mathcal{A} = \mathbb{e}^{\alpha}$, and $\bar{\mathcal{A}} = \mathcal{A}^{-1}$, we have

$$\tilde{R}_{ij} = \mathbb{e}^{\hbar(y_i x_j - t_i a_j)}\left(1 + \epsilon\hbar\left(a_i a_j - \hbar^2 y_i^2 x_j^2/4\right) + O(\epsilon^2)\right)$$

in $\mathcal{S}(B_i, B_j)$, and in $\mathcal{S}(B_1^*, B_2^*, B)$ we have

$$\tilde{m} = \mathbb{e}^{(\alpha_1+\alpha_2)a + \eta_2\xi_1(1-T)/\hbar + (\xi_1\bar{\mathcal{A}}_2 + \xi_2)x + (\eta_1 + \eta_2\bar{\mathcal{A}}_1)y}\left(1 + \epsilon\lambda + O(\epsilon^2)\right),$$

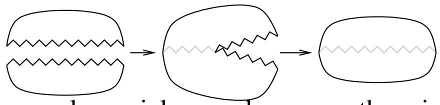
where $\lambda = 2a\eta_2\xi_1 T + \eta_2^2\xi_1^2\left(3T^2 - 4T + 1\right)/4\hbar - \eta_2\xi_1^2(3T-1)x\bar{\mathcal{A}}_2/2$
$-\eta_2^2\xi_1(3T-1)y\bar{\mathcal{A}}_1/2 + \eta_2\xi_1 xy\hbar\bar{\mathcal{A}}_1\bar{\mathcal{A}}_2$.

Finally,

$$\tilde{\Delta} = \mathbb{e}^{\tau(t_1+t_1) + \eta(y_1 + T_1 y_2) + \alpha(a_1+a_2) + \xi(x_1+x_2)}\left(1 + O(\epsilon)\right) \in \mathcal{S}(B^*, B_1, B_2),$$

and $\tilde{S} = \mathbb{e}^{-\tau t - \alpha a - \eta\xi(1-\bar{T})\mathcal{A}/\hbar - \bar{T}\eta y\mathcal{A} - \xi x\mathcal{A}}\left(1 + O(\epsilon)\right) \in \mathcal{S}(B^*, B)$.

The Zipping Issue. (between unbound and bound lies half-zipped).



Zipping. If $P(\zeta^j, z_i)$ is a polynomial, or whenever otherwise convergent, set $\left\langle P(\zeta^j, z_i)\right\rangle_{(\zeta^j)} = P\left(\partial_{z_j}, z_i\right)\Big|_{z_i=0}$. (E.g., if $P = \sum a_{nm}\zeta^n z^m$ then $\langle P\rangle_\zeta = \sum a_{nm}\,\partial_z^n z^m\big|_{z=0} = \sum n! a_{nn}$).

The Zipping / Contraction Theorem. If $P = P(\zeta^j, z_i)$ has a finite ζ-degree and the y's and the q's are "small" then

$$\left\langle P\mathbb{e}^{c + \eta^i z_i + y_j\zeta^j + q_j^i z_i\zeta^j}\right\rangle_{(\zeta^j)} = \det(\tilde{q})\mathbb{e}^{c + \eta^i\tilde{q}_i^k y_k}\left\langle P\,\middle|\,{\zeta^j \to \zeta^j + \eta^i\tilde{q}_i^j \atop z_i \to \tilde{q}_i^k(z_k + y_k)}\right\rangle_{(\zeta^j)}$$

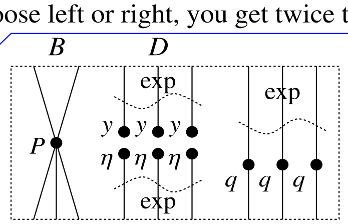
where $\tilde{q}$ is the inverse matrix of $1 - q$: $(\delta_j^i - q_j^i)\tilde{q}_k^j = \delta_k^i$.

Exponential Reservoirs. The true Hilbert hotel is exp! Remove one x from an "exponential reservoir" of x's and you are left with the same exponential reservoir:

$$\mathbb{e}^x = \left[\ldots + \frac{xxxxx}{120} + \ldots\right] \xrightarrow{\partial_x} \left[\ldots + \frac{x\not{x}xxx}{120} + \ldots\right] = (\mathbb{e}^x)' = \mathbb{e}^x,$$

and if you let each element choose left or right, you get twice the same reservoir:

$$\mathbb{e}^x \xrightarrow{x \to x_l + x_r} \mathbb{e}^{x_l + x_r} = \mathbb{e}^{x_l}\mathbb{e}^{x_r}.$$



A Graphical Proof. Glue top to bottom on the right, in all possible ways. Several scenarios occur:

1. Start at A, go through the q-machine $k \geq 0$ times, stop at B. Get $\left\langle P\left(\zeta, \sum_{k\geq 0} q^k z\right)\right\rangle = \langle P(\zeta, \tilde{q}z)\rangle$.
2. Loop through the q-machine and swallow your own tail. Get $\exp\left(\sum q^k/k\right) = \exp(-\log(1-q)) = \tilde{q}$. 3. ...

By the reservoir splitting principle, these scenarios contribute multiplicatively. □

Implementation. ($\mathbb{E}[Q, P]$ means $\mathbb{e}^Q P$) ωεβ/Zip

```
Zip_{ζs_List}@𝔼[Q_, P_] :=
  Module[{ζ, z, zs, c, ys, ηs, qt, zrule, ζrule},
   zs = Table[ζ*, {ζ, ζs}];
   c = Q /. Alternatives @@ (ζs ⋃ zs) → 0;
   ys = Table[∂_ζ (Q /. Alternatives @@ zs → 0), {ζ, ζs}];
   ηs = Table[∂_z (Q /. Alternatives @@ ζs → 0), {z, zs}];
   qt = Inverse@Table[Kδ_{z,ζ*} - ∂_{z,ζ}Q, {ζ, ζs}, {z, zs}];
   zrule = Thread[zs → qt.(zs + ys)];
   ζrule = Thread[ζs → ζs + ηs.qt];
   Simplify /@
    𝔼[c + ηs.qt.ys, Det[qt] Zip_{ζs}[P /. (zrule ⋃ ζrule)]] ];
```

Real Zipping is a minor mess, and is done in two phases:

	τa-phase		ξy-phase	
ζ-like variables	τ	a	ξ	y
z-like variables	t	α	x	η

Already at $\epsilon = 0$ we get the best known formulas for the Alexander polynomial!

Generic Docility. A "docile perturbed Gaussian" in the variables $(z_i)_{i\in S}$ over the ring R is an expression of the form

$$\mathbb{e}^{q^{ij}z_i z_j}P = \mathbb{e}^{q^{ij}z_i z_j}\left(\sum_{k\geq 0}\epsilon^k P_k\right),$$

where all coefficients are in R and where P is a "docile series": $\deg P_k \leq 4k$.

Our Docility. In the case of QU_ϵ, all invariants and operations are of the form $\mathbb{e}^{L+Q}P$, where

- L is a quadratic of the form $\sum l_{z\zeta}z\zeta$, where z runs over $\{t_i, \alpha_i\}_{i\in S}$ and ζ over $\{\tau_i, a_i\}_{i\in S}$, with integer coefficients $l_{z\zeta}$.
- Q is a quadratic of the form $\sum q_{z\zeta}z\zeta$, where z runs over $\{x_i, \eta_i\}_{i\in S}$ and ζ over $\{\xi_i, y_i\}_{i\in S}$, with coefficients $q_{z\zeta}$ in the ring R_S of rational functions in $\{T_i, \mathcal{A}_i\}_{i\in S}$.
- P is a docile power series in $\{y_i, a_i, x_i, \eta_i, \xi_i\}_{i\in S}$ with coefficients in R_S, and where $\deg(y_i, a_i, x_i, \eta_i, \xi_i) = (1, 2, 1, 1, 1)$.

Docililty Matters! The rank of the space of docile series to ϵ^k is polynomial in the number of variables $|S|$. !!!!!

- At $\epsilon^2 = 0$ we get the Rozansky-Overbay [Ro1, Ro2, Ro3, Ov] invariant, which is stronger than HOMFLY-PT polynomial and Khovanov homology taken together!
- In general, get "higher diagonals in the Melvin-Morton-Rozansky expansion of the coloured Jones polynomial" [MM, BNG], but why spoil something good?

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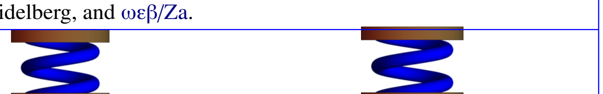

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

www.katlas.org

Video and more at http://www.math.toronto.edu/~drorbn/Talks/Ohio-1901