

```

Define[op_is_] := 
Module[{SD, ii, jj, kk, isp, nis, nisp, sis},
Block[{i, j, k},
ReleaseHold[Hold[
SD[opnisp,$k_Integer, Block[{i, j, k}, opisp,$k = 8;
opnis,sk]];
SD[opisp, op{is},$k]; SD[opsis_, op{sis}]];
/. {SD → SetDelayed,
isp → {is}} /. {i → i_, j → j_, k → k_},
nis → {is}} /. {i → ii_, j → jj_, k → kk_},
nisp → {is}} /. {i → ii_, j → jj_, k → kk_}
}]]

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## The Fundamental Tensors

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Define[ami,j,k = E{i,j}→{k}[(αi+αj) ak, (e-y aj εi+εj) xk, 1]k,
bmi,j,k = E{i,j}→{k}[(βi+βj) bk, (ηi+ηj) yk, e(e^-ε βi-1) ηj yk]k]
Define[Ri,j =
E{i,j}→{i,j}[h aj bi, h xj yi, e^((1-ey h)k (h yi xj)k)]k]
Define[barRi,j = E{i,j}→{i,j}[-h aj bi, -h xj yi/bi,
1+If[$k == 0, 0, (barRi,j, $k-1)k[3] -
((barRi,j, 0)k R1,2 (barR{3,4}, $k-1)k) // (bmi,1→i amj,2→j) //
(bmi,3→i amj,4→j) [3]]], 
Pi,j = E{i,j}→{i}[(βi αj / h, ηi εj / h,
1+If[$k == 0, 0, (P{i,j},0, $k-1)k[3] -
(R1,2 // ((P{i,j},0, P{i,2}, $k-1)k))[3]]]]
Define[aSj = barRi,j~Bi~Pi,j,
aSi = E{i}→{i}[-ai αi, -xi εi εi,
1+If[$k == 0, 0, (aSi, $k-1)k[3] -
((aSi, 0)k~Bi~aSi~Bi~Pi,j~(aSi, $k-1)k)[3]]]]
Define[bSi = Ri,1~B1~aS1~B1~Pi,1,
barbSi = Ri,1~B1~aSi~B1~Pi,1,
aΔi,j,k = (Ri,j R2,k) // bmi,2→3 // P3,i,
bΔi,j,k = (Rj,1 Rk,2) // ami,2→3 // Pi,3]

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Define[
dmi,j,k =
(E{i,j}→{i,j}[(βi bi + αj aj, ηi yi + εj xj, 1]
(aΔi+1,2 // aΔ2→2,3 // aS3) (bΔj+1,-2 // bΔ-2→-2,-3) // 
(P1,3 P-3,1 am2,j→k bm1,-2+k),
dSi = E{i}→{1,2}[(βi bi + αi a2, ηi y1 + εi x2, 1) // (bS1 aS2) // 
dm2,1→i,
dΔi,j,k = (bΔi+3,1 aΔi+2,4) // (dm3,4→k dm1,2→j) ]
Define[Ci = E{i}→{i}[(0, 0, Bi1/2 e-h ε ai/2)]k,
C̄i = E{i}→{i}[(0, 0, Bi-1/2 eh ε ai/2)]k,
Kinki = (R1,3 C2) // dm1,2→1 // dm1,3→i,
Kink̄i = (R̄1,3 C2) // dm1,2→1 // dm1,3→i]

```

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Define[
b2ti = E{i}→{i}[(αi ai - βi ti / γ, εi xi + ηi yi, eε βi ai/γ)]k,
t2bi = E{i}→{i}[(αi ai - ti γ bi, εi xi + ηi yi, eε ti ai)]k
Define[kRi,j = Ri,j // (b2ti b2tj) /. ti|j → t,
kR̄i,j = R̄i,j // (b2ti b2tj) /. ti|j → T,
kmi,j,k = (t2bi t2bj) // dmi,j,k // 
b2tk /. {tk → t, Tk → T, ti|j → 0},
kCi = Ci // b2ti /. Ti → T, kC̄i = C̄i // b2ti /. Ti → T,
kKinki = Kinki // b2ti /. {ti → t, Ti → T},
kKink̄i = Kink̄i // b2ti /. {ti → t, Ti → T}]

```

## The Trefoil

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$k = 2; Z = kR1,5 kR6,2 kR3,7 kC̄4 kKink8 kKink9 kKink10;
Do[Z = Z~B1,r~km1,r→1, {r, 2, 10}];
Simplify@Z /. v_1 → v
E{i}→{1}[(0, 0, T / (1 - T + T2) + 1 / (1 - T + T2)3 T h (2 a (-1 + T - T3 + T4) +
T (-1 + 2 T - 3 T2 + 2 T3) γ - 2 (1 + T3) x y γ h) ∈ +
1 / (1 - T + T2)5 T h2 (4 a2 (1 - T + T2)2 (1 + T - 6 T2 + T3 + T4) +
4 a (1 - T + T2) γ (T (2 - 5 T + 8 T2 - 7 T3 - 2 T4 + 2 T5) -
2 (-1 - 2 T + 5 T2 - 4 T3 + T4 + 2 T5) x y h) +
γ2 (T (1 - 2 T + 4 T2 - 2 T3 + 6 T5 - 11 T6 + 4 T7) +
4 (-1 + 2 T + T3 + T4 + 2 T6 - T7) x y h +
6 (1 - T + T2)2 (1 + 3 T + T2) x2 y2 h2) ) ∈2 + O[ε]3]

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diagram	$n_k^i$	Alexander's $\omega^+$ genus / ribbon	diagram	$n_k^i$	Alexander's $\omega^+$ genus / ribbon	diagram	$n_k^i$	Alexander's $\omega^+$ genus / ribbon
	Today's $\rho_1^+$	unknotting # / amphi?		Today's $\rho_1^+$	unknotting # / amphi?		Today's $\rho_1^+$	unknotting # / amphi?
	0 <sup>a</sup> <sub>1</sub>	1		3 <sup>a</sup> <sub>1</sub>	$t - 1$		1 / X	
	0	0 / ✓		t			4 <sup>a</sup> <sub>1</sub>	3 - t
	5 <sup>a</sup> <sub>1</sub>	$t^2 - t + 1$		5 <sup>a</sup> <sub>2</sub>	$2t - 3$		1 / X	
	$2t^3 + 3t$	2 / X		5 <sup>a</sup> <sub>3</sub>	$t - 4$		0	1 / ✓
	6 <sup>a</sup> <sub>2</sub>	$-t^2 + 3t - 3$		6 <sup>a</sup> <sub>3</sub>	$t^2 - 3t + 5$		6 <sup>a</sup> <sub>4</sub>	$5 - 2t$
	$t^3 - 4t^2 + 4t - 4$	1 / X		0			t - 4	1 / X
	7 <sup>a</sup> <sub>2</sub>	$3t - 5$		7 <sup>a</sup> <sub>3</sub>	$2t^2 - 3t + 3$		7 <sup>a</sup> <sub>4</sub>	$4t - 7$
	$14t - 16$	1 / X		-9t <sup>3</sup> + 8t <sup>2</sup> - 16t + 12			32 - 24t	2 / X
	7 <sup>a</sup> <sub>5</sub>	$2t^2 - 4t + 5$		7 <sup>a</sup> <sub>6</sub>	$-t^2 + 5t - 7$		7 <sup>a</sup> <sub>7</sub>	$t^2 - 5t + 9$
	$9t^3 - 16t^2 + 29t - 28$	2 / X		t <sup>3</sup> - 8t <sup>2</sup> + 19t - 20			8 - 3t	1 / X
	8 <sup>a</sup> <sub>1</sub>	$7 - 3t$		8 <sup>a</sup> <sub>2</sub>	$-t^3 + 3t^2 - 3t + 3$		8 <sup>a</sup> <sub>3</sub>	$9 - 4t$
	$5t - 16$	1 / X		2t <sup>5</sup> - 8t <sup>4</sup> + 10t <sup>3</sup> - 12t <sup>2</sup> + 13t - 12	2 / X		0	2 / ✓
	8 <sup>a</sup> <sub>4</sub>	$-2t^2 + 5t - 5$		8 <sup>a</sup> <sub>5</sub>	$-t^3 + 3t^2 - 4t + 5$		8 <sup>a</sup> <sub>6</sub>	$-2t^2 + 6t - 7$
	$3t^3 - 8t^2 + 6t - 4$	2 / X		-2t <sup>5</sup> + 8t <sup>4</sup> - 13t <sup>3</sup> + 20t <sup>2</sup> - 22t + 24	2 / X		$5t^3 - 20t^2 + 28t - 32$	2 / X
	8 <sup>a</sup> <sub>7</sub>	$t^3 - 3t^2 + 5t - 5$		8 <sup>a</sup> <sub>8</sub>	$2t^2 - 6t + 9$		8 <sup>a</sup> <sub>9</sub>	$t^2 - 5t + 7$
	$-t^5 + 4t^4 - 10t^3 + 12t^2 - 13t + 12$	1 / X		-t <sup>3</sup> + 4t <sup>2</sup> - 12t + 16	2 / X		0	1 / ✓
	8 <sup>a</sup> <sub>10</sub>	$t^3 - 3t^2 + 6t - 7$		8 <sup>a</sup> <sub>11</sub>	$-2t^2 + 7t - 9$		8 <sup>a</sup> <sub>12</sub>	$t^2 - 7t + 13$
	$-t^5 + 4t^4 - 11t^3 + 16t^2 - 21t + 20$	2 / X		5t <sup>3</sup> - 24t <sup>2</sup> + 39t - 44	1 / X		0	2 / ✓
	8 <sup>a</sup> <sub>13</sub>	$2t^2 - 7t + 11$		8 <sup>a</sup> <sub>14</sub>	$-2t^2 + 8t - 11$		8 <sup>a</sup> <sub>15</sub>	$3t^2 - 8t + 11$
	$-t^3 + 4t^2 - 14t + 20$	1 / X		5t <sup>3</sup> - 28t <sup>2</sup> + 57t - 68	1 / X		$21t^3 - 64t^2 + 120t - 140$	2 / X
	8 <sup>a</sup> <sub>16</sub>	$t^3 - 4t^2 + 8t - 9$		8 <sup>a</sup> <sub>17</sub>	$-t^3 + 4t^2 - 8t + 11$		8 <sup>a</sup> <sub>18</sub>	$t^3 + 5t^2 - 10t + 13$
	$t^5 - 6t^4 + 17t^3 - 28t^2 + 35t - 36$	2 / X		0	1 / ✓		0	2 / ✓
	8 <sup>a</sup> <sub>19</sub>	$t^3 - t^2 + 1$		8 <sup>a</sup> <sub>20</sub>	$t^2 - 2t + 3$		8 <sup>a</sup> <sub>21</sub>	$-t^2 + 4t - 5$
	$-3t^5 - 4t^2 - 3t$	3 / X		4t - 4	1 / X		$t^3 - 8t^2 + 16t - 20$	1 / X

Video and more at <http://www.math.toronto.edu/~drorbn/Talks/Ohio-1901>