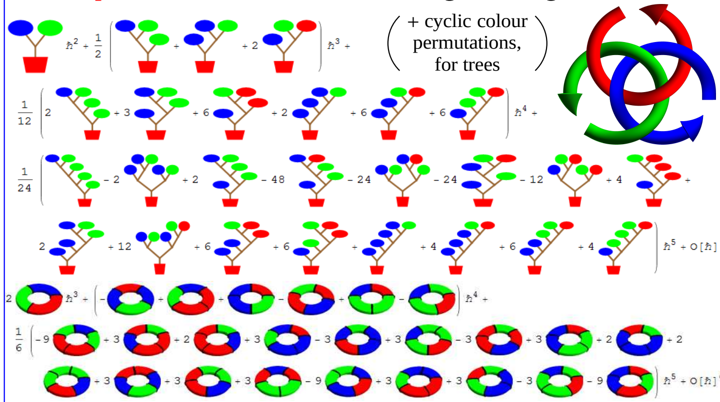
Polynomial Time Knot Polynomials, B

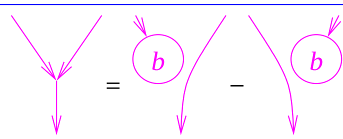
Theorem 2 [BND]. ∃! a homomorphic expansion, aka a homomorphic universal finite type invariant Z^w of pure w-tangles. $z^w := \log Z^w$ takes values in $FL(S)^S \times CW(S)$.

z is computable. z of the Borromean tangle, to degree 5 [BN]:



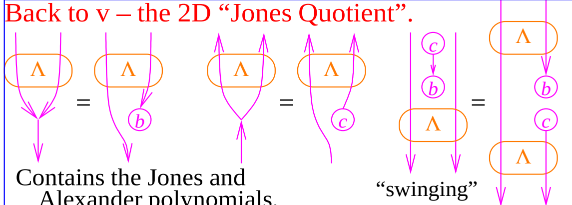
(I have a fancy free-Lie calculator!) (ωεβ/FLD) Nice, but too hard!

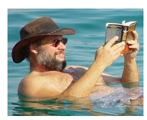
Proposition [BN]. Modulo all relations that universally hold for the 2D non-Abelian Lie algebra and after some changes-of-variable, z^w reduces to z_0.



$$[u,v] = b_u v - b_v u$$

Back to v – the 2D "Jones Quotient".

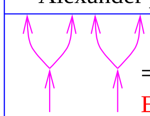




V. Jones

Contains the Jones and Alexander polynomials, "swinging" ...yet still too hard!

The OneCo Quotient. Likely related to [ADO]



= 0, only one co-bracket is allowed.

Everything should work, and everything is being worked!

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Help Needed!

I'm slow and feeble-minded.

The Abstract Context

Definition. (Compare [BNS, BN]) A meta-monoid is a functor M: (finite sets, injections)→(sets) (think "$M(S)$ is quantum G^S", for G a group) along with natural operations $*\colon M(S_1)\times M(S_2)\to M(S_1\sqcup S_2)$ whenever $S_1\cap S_2=\emptyset$ and $m^{ab}_c\colon M(S)\to M((S\setminus\{a,b\})\sqcup\{c\})$ whenever $a\neq b\in S$ and $c\notin S\setminus\{a,b\}$, such that

$$\text{meta-associativity:}\quad m^{ab}_a /\!\!/ m^{ac}_a = m^{bc}_b /\!\!/ m^{ab}_a$$

$$\text{meta-locality:}\quad m^{ab}_c /\!\!/ m^{de}_f = m^{de}_f /\!\!/ m^{ab}_c$$

and, with $\epsilon_b = M(S \hookrightarrow S\sqcup\{b\})$,

$$\text{meta-unit:}\quad \epsilon_b /\!\!/ m^{ab}_a = Id = \epsilon_b /\!\!/ m^{ba}_a.$$

Claim. Pure virtual tangles $\mathcal{PVT}$ form a meta-monoid.

Theorem. $S\mapsto \Gamma_0(S)$ is a meta-monoid and $z_0\colon \mathcal{PVT}\to\Gamma_0$ is a morphism of meta-monoids.

Strong Conviction. There exists an extension of Γ_0 to a bigger meta-monoid $\Gamma_{01}(S)=\Gamma_0(S)\times\Gamma_1(S)$, along with an extension of z_0 to $z_{01}\colon\mathcal{PVT}\to\Gamma_{01}$, with

$$\Gamma_1(S) = V\oplus V^{\otimes 2}\oplus V^{\otimes 3}\oplus \mathcal{S}^2(V)^{\otimes 2}\qquad (\text{with } V := R_S\langle S\rangle).$$

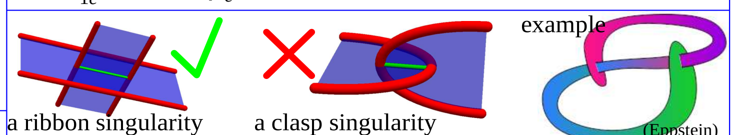
Furthermore, upon reducing to a single variable everything is polynomial size and polynomial time.

Furthermore, Γ_{01} is given using a "meta-2-cocycle ρ^{ab}_c over Γ_0": In addition to $m^{ab}_c \to m^{ab}_{0c}$, there are R_S-linear $m^{ab}_{1c}\colon\Gamma_1(S\sqcup\{a,b\})\to\Gamma_1(S\sqcup\{c\})$, a meta-right-action $\alpha^{ab}\colon\Gamma_1(S)\times\Gamma_0(S)\to\Gamma_1(S)$ R_S-linear in the first variable, and a first order differential operator (over R_S) $\rho^{ab}_c\colon\Gamma_0(S\sqcup\{a,b\})\to\Gamma_1(S\sqcup\{c\})$ such that

$$(\zeta_0,\zeta_1)/\!\!/ m^{ab}_c = \left(\zeta_0 /\!\!/ m^{ab}_{0c}, (\zeta_1,\zeta_0)/\!\!/\alpha^{ab}/\!\!/ m^{ab}_{1c} + \zeta_0/\!\!/\rho^{ab}_c\right)$$

What's done? The braid part, with still-ugly formulas.

What's missing? A lot of concept- and detail-sensitive work towards m^{ab}_{1c}, α^{ab}, and ρ^{ab}_c. The "ribbon element".



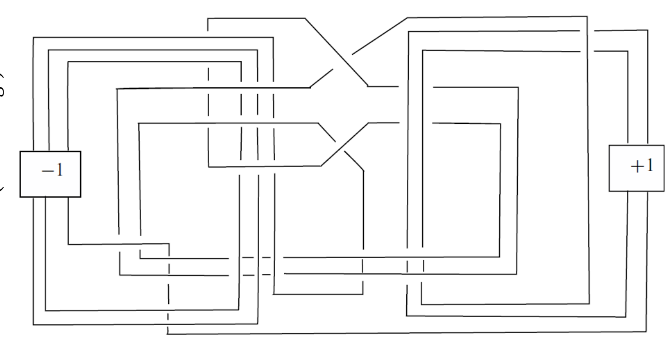
a ribbon singularity a clasp singularity

A bit about ribbon knots. A "ribbon knot" is a knot that can be presented as the boundary of a disk that has "ribbon singularities", but no "clasp singularities". A "slice knot" is a knot in $S^3=\partial B^4$ which is the boundary of a non-singular disk in B^4. Every ribbon knots is clearly slice, yet,

Conjecture. Some slice knots are not ribbon.

Fox-Milnor. The Alexander polynomial of a ribbon knot is always of the form $A(t)=f(t)f(1/t)$. (also for slice)

[GST]: a slice knot that might not be ribbon (48 crossings).



"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

www.katlas.org

Video and more at http://www.math.toronto.edu/~drorbn/Talks/LesDiablerets-1508/