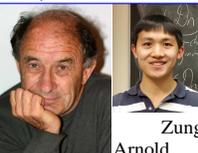


Finite Type Invariants of Doodles, 1



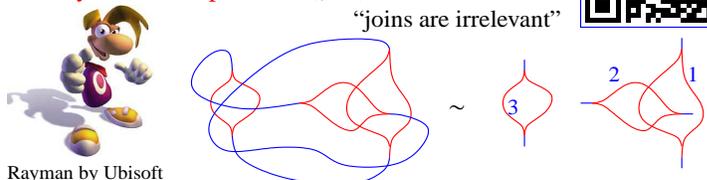
Abstract. I will describe my former student's Jonathan Zung work on finite type invariants of "doodles", plane curves modulo the second Reidemeister move but not modulo the third. We use a definition of "finite type" different from Arnold's and more along the lines of Goussarov's "Interdependent Modifications", and come to a conjectural combinatorial description of the set of all such invariants. We then describe how to construct many such invariants (though perhaps not all) using a certain class of 2-dimensional "configuration space integrals".

An unfinished project!

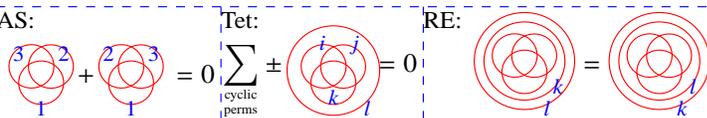
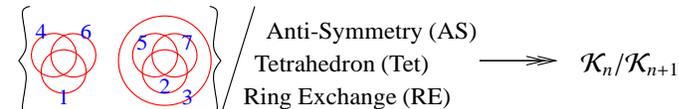
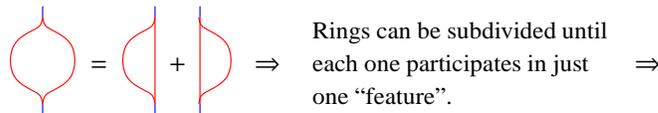


Chord Diagrams and an Upper Bound on $\mathcal{K}_n/\mathcal{K}_{n+1}$

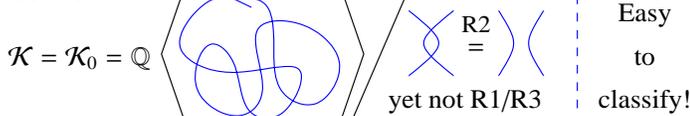
The Rayman Principle. In $\mathcal{K}_n/\mathcal{K}_{n+1}$, "joins are irrelevant"



The Subdivision Relations. In $\mathcal{K}_n/\mathcal{K}_{n+1}$,



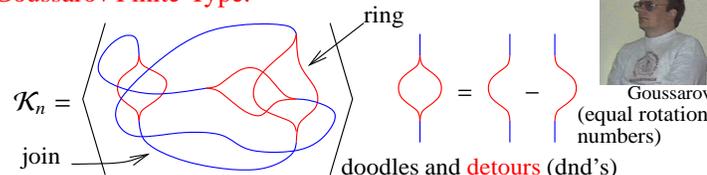
Doodles.



Prior Art. Arnold [Ar] first studied doodles within his study of plane curves and the "strangeness" St invariant. Vassiliev [Va1, Va2] defined finite type invariants in a different way, and Merkov [Me] proved that they separate doodles.



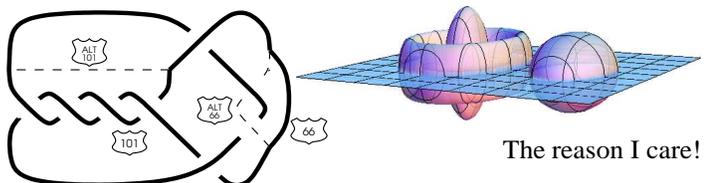
Goussarov Finite-Type.



Def. V is of type n if it vanishes on \mathcal{K}_{n+1} . $(\mathcal{K}_0/\mathcal{K}_{n+1})^* \leftrightarrow \mathcal{K}_n/\mathcal{K}_{n+1}$

Knots in 3D.

2-Knots in 4D.



Goals. • Describe $\mathcal{A}_n := \mathcal{K}_n/\mathcal{K}_{n+1}$ using diagrams/relations. • Get many or all finite type invariants of doodles using configurations space integrals. • Do these come from a TQFT? • See if \mathcal{A}_n has a "Lie theoretic" (tensors/relations) meaning. • See if/how Arnold's St and the Merkov invariants integrate in.

The Primary Snippet.

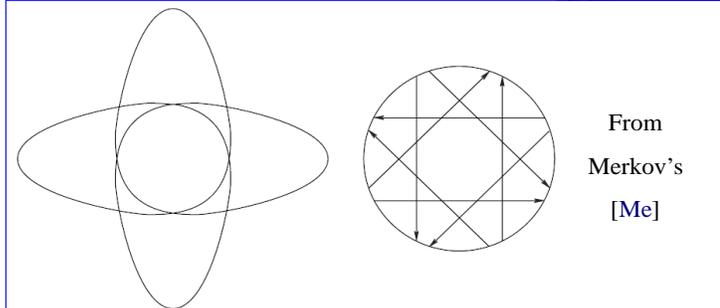
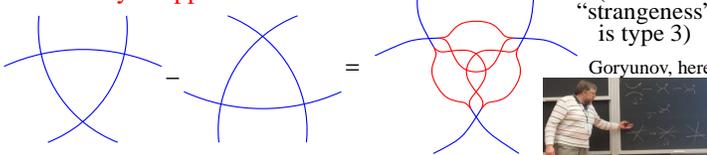
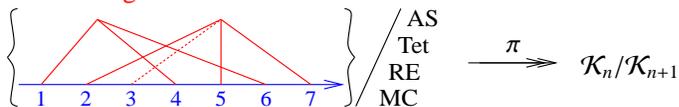
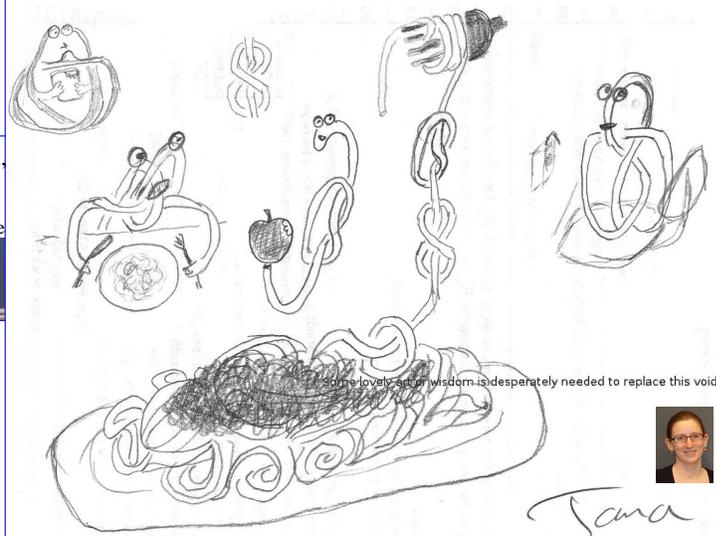
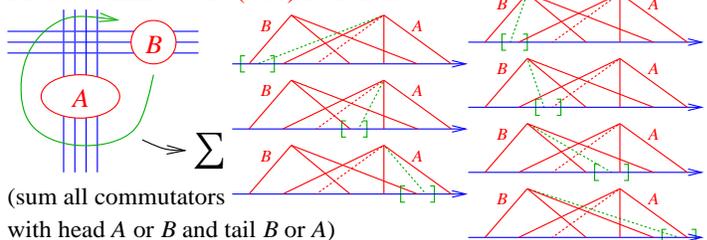


Figure 3. A non-trivial 1-doodle and its arrow diagram

"Chord Diagrams".



"Multi-Commutator" (MC) Relations.



Doodles by my former student Jana Archibald



"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

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