The 17 Tiling Patterns: *Gotta catch 'em all!*

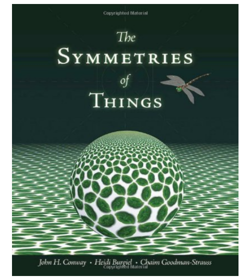
Treehouse Talks, Friday October 17, 2014, Beeton Auditorium, Toronto Reference Library, 789 Yonge Street, 6:30PM

Abstract. My goal is to get you hooked, captured and unreleased until you find all 17 in real life, around you.

We all know know that the plane can be filled in different periodic manners: floor tiles are often square but sometimes hexagonal, bricks are often laid in an interlaced pattern, fabrics often carry interesting patterns. A little less known is that there are precisely 17 symmetry patterns for tiling the plane; not one more, not one less. It is even less known how easy these 17 are to identify in the patterns around you, how fun it is, how common some are, and how rare some others seem to be.

Gotta catch 'em all!

Reading. An excellent book on the subject is *The Symmetries of Things* by J. H. Conway, H. Burgiel, and C. Goodman-Strauss, CRC Press, 2008. Another nice text is *Classical Tessellations and Three-Manifolds* by J. M. Montesinos, Springer-Verlag, 1987.



Question. In what ways can you make \$2 change, using coins denominated $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, etc.?

Answer. $2=\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{3}{4}+\frac{3}{4}+\frac{1}{2}=\frac{5}{6}+\frac{2}{3}+\frac{1}{2}$, and that's it.

Video, handout, links at drorbn.net/Treehouse

The Basic Features.

Gotta catch 'em all!

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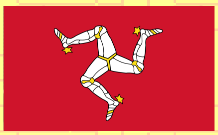


rotation only

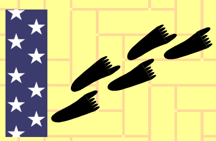
3⃒

rotation-reflection

M

free mirror-reflection

G

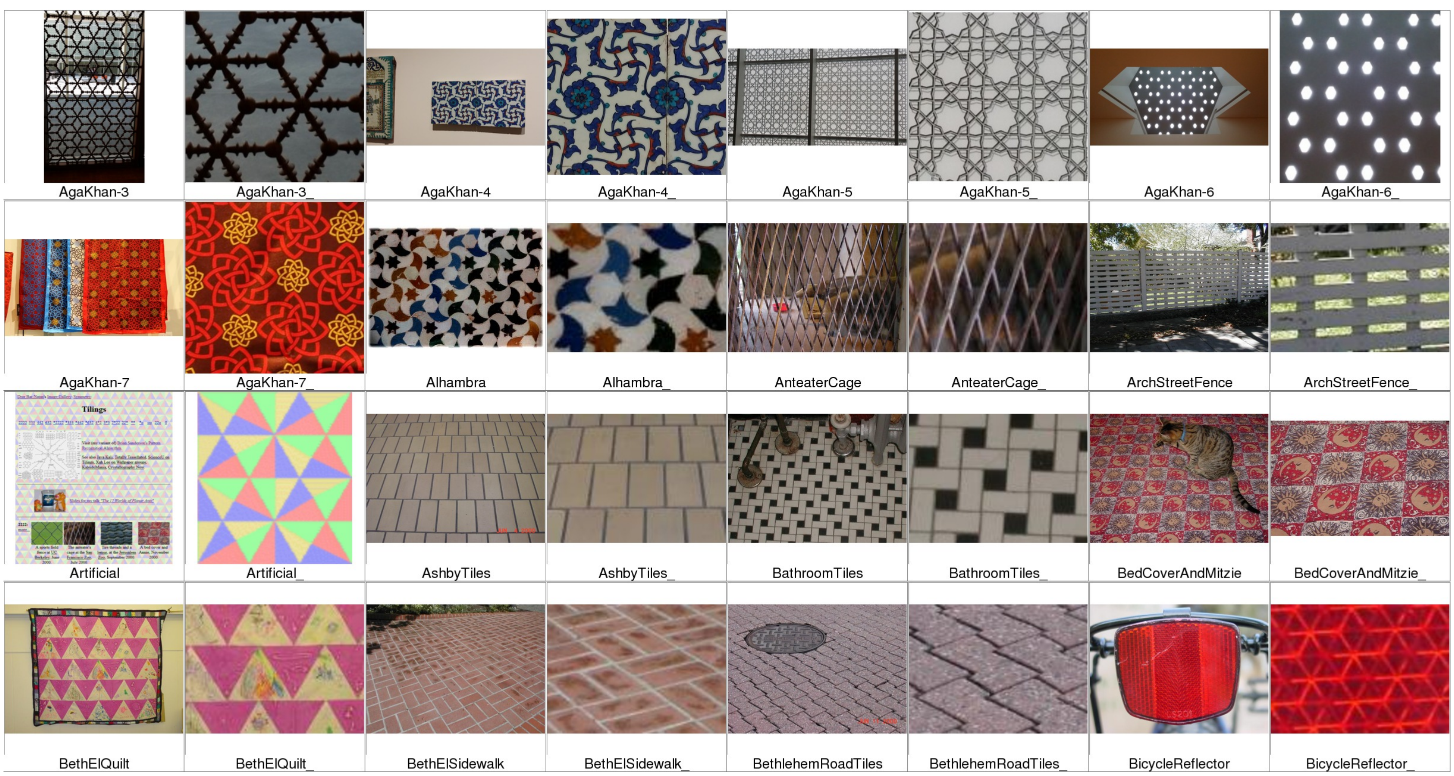


free glide-reflection

Theorem. There are precisely 17 patterns with which to tile the plane, no more, no less. They are all made of combinations of the 10 basic features, 2, 3, 4, 6, 2⃒, 3⃒, 4⃒, 6⃒, M, and G, as follows:

✓	Dror's	Conway's	crystallo -graphic	✓	Dror's	Conway's	crystallo -graphic
	2222	2222	p2		33⃒	3*3	p31m
	333	333	p3		22⃒2⃒	2*22	cmm
	442	442	p4		22M	22*	pmg
	632	632	p6		MM	**	pm
	2⃒2⃒2⃒2⃒	*2222	pmm		MG	*o	cm
	3⃒3⃒3⃒	*333	p3m1		GG	oo	pg
	4⃒4⃒2⃒	*442	p4m		22G	22o	pgg
	6⃒3⃒2⃒	*632	p6m		∅	0	p1
	42⃒	4*2	p4g				

Tilings worksheet. Classify the following pictures according to the following possibilities: **2222**=2222, **333**=333, **442**=442, **632**=632, **2⃒2⃒2⃒2⃒**=*2222, **3⃒3⃒3⃒**=*333, **4⃒4⃒2⃒**=*442, **6⃒3⃒2⃒**=*632, **42⃒**=4*2, **33⃒**=3*3, **22⃒2⃒**=2*22, **22M**=22*, **MM**=**, **MG**=*o, **GG**=oo, **22G**=22o, and **∅**=0 (the pictures come in {context, pattern} pairs).



AgaKhan-3, AgaKhan-3_, AgaKhan-4, AgaKhan-4_, AgaKhan-5, AgaKhan-5_, AgaKhan-6, AgaKhan-6_

AgaKhan-7, AgaKhan-7_, Alhambra, Alhambra_, AnteaterCage, AnteaterCage_, ArchStreetFence, ArchStreetFence_

Artificial, Artificial_, AshbyTiles, AshbyTiles_, BathroomTiles, BathroomTiles_, BedCoverAndMitzie, BedCoverAndMitzie_

BethElQuilt, BethElQuilt_, BethElSidewalk, BethElSidewalk_, BethlehemRoadTiles, BethlehemRoadTiles_, BicycleReflector, BicycleReflector_

Video and more at http://www.math.toronto.edu/~drorbn/Talks/Treehouse-1410/