

Abstract. To break a week of deep thinking with a nice colourful light dessert, we will present the Kolmogorov-Arnold solution of Hilbert's 13th problem with lots of computer-generated rainbow-painted 3D pictures.

In short, Hilbert asked if a certain specific function of three variables can be written as a multiple (yet finite) composition of continuous functions of just two variables. Kolmogorov and Arnold showed him silly (ok, it took about 60 years, so it was a bit tricky) by showing that **any** continuous function f of any finite number of variables is a finite composition of continuous functions of a single variable and several instances of the binary function "+" (addition). For $f(x, y) = xy$, this may be $xy = \exp(\log x + \log y)$. For $f(x, y, z) = x^y/z$, this may be $\exp(\exp(\log y + \log \log x) + (-\log z))$. What might it be for (say) the real part of the Riemann zeta function?

The only original material in this talk will be the pictures; the math was known since around 1957.



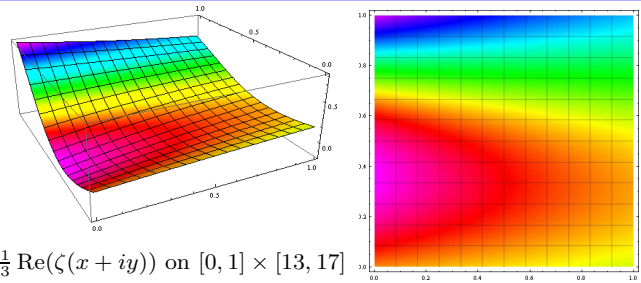
Hilbert



Kolmogorov



Arnold (by Moser)



$\frac{1}{3} \operatorname{Re}(\zeta(x + iy))$ on $[0, 1] \times [13, 17]$

Fix an irrational $\lambda > 0$, say $\lambda = (\sqrt{5} - 1)/2$. All functions are continuous.

Theorem. There exist five $\phi_i : [0, 1] \rightarrow [0, 1]$ ($1 \leq i \leq 5$) so that for every $f : [0, 1] \times [0, 1] \rightarrow \mathbb{R}$ there exists a $g : [0, 1 + \lambda] \rightarrow \mathbb{R}$ so that

$$f(x, y) = \sum_{i=1}^5 g(\phi_i(x) + \lambda\phi_i(y))$$

for every $x, y \in [0, 1]$.

Step 1. If $\epsilon > 0$ and $f : [0, 1] \times [0, 1] \rightarrow \mathbb{R}$, then there exists $\phi : [0, 1] \rightarrow [0, 1]$ and $g : [0, 1 + \lambda] \rightarrow \mathbb{R}$ so that $|f(x, y) - g(\phi(x) + \lambda\phi(y))| < \epsilon$ on at least 98% of the area of $[0, 1] \times [0, 1]$.

The key. "Poorify" chocolate bars.

