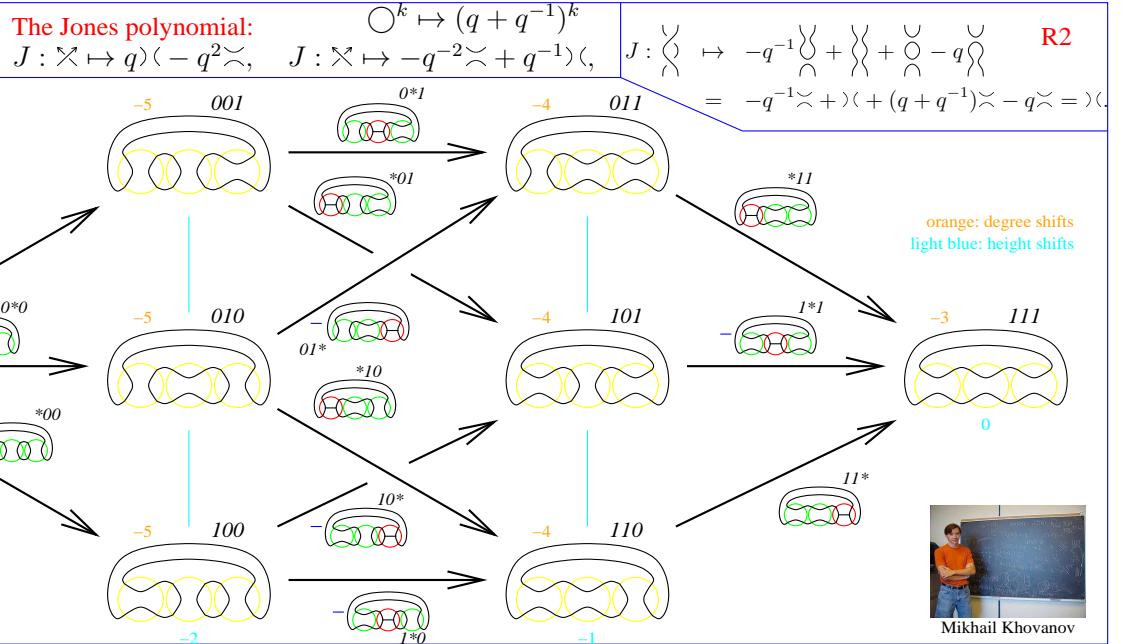


# Local Khovanov Homology (1)

(an outdated overview)

**What is it?**

A cube for each knot/link projection;

Vertices: All fillings of with or with .

Edges: All fillings of  $I \times \langle \rangle$  = with  $I \times \langle \rangle$  = or with  $I \times \langle \rangle$  = and precisely one .**Signs?**

$$\begin{array}{ccccc} dx & \xrightarrow{+} & dx \wedge dy & & \\ \swarrow \begin{smallmatrix} \wedge dx \\ \wedge dy \\ \wedge dz \end{smallmatrix} & & \searrow \begin{smallmatrix} + \\ - \\ + \end{smallmatrix} & & \\ I & \xrightarrow{+} & dx \wedge dz & \xrightarrow{+} & \\ \searrow \begin{smallmatrix} + \\ - \\ - \end{smallmatrix} & & \swarrow \begin{smallmatrix} + \\ - \end{smallmatrix} & & \\ dy & \xrightarrow{-} & dy \wedge dz & & \end{array}$$

**More crossings?****General Crossings**

$$\begin{array}{c} \text{Crossing} \rightarrow \left( \text{Crossing} \xrightarrow{+1} \text{Crossing} \right) \\ \text{Crossing} \rightarrow \left( \text{Crossing} \xrightarrow{-2} \text{Crossing} \right) \end{array}$$

**Where does it live?**In  $\text{Kom}(\text{Mat}(\langle \text{Cob} \rangle / \{S, T, G, NC\})) / \text{homotopy}$ **Kom:** Complexes   **Mat:** Matrices**Cob:** Cobordisms  $\langle \dots \rangle$ : Formal lin. comb.**Cob:**

$$\text{Diagram showing } \circ = \text{cylinder with boundary} = \text{cylinder with boundary}$$

**Mat(C):**

$$\begin{array}{ccc} \left( \begin{array}{c} O'_1 \\ O''_1 \\ O'_2 \\ O''_2 \end{array} \right) & \xrightarrow{\begin{array}{c} G_{11} \\ G_{21} \\ G_{31} \\ G_{12} \end{array}} & \left( \begin{array}{c} O'_1 \\ O''_1 \\ O'_2 \\ O''_2 \end{array} \right) \\ & \xrightarrow{\begin{array}{c} F_{21} \\ F_{22} \\ F_{23} \end{array}} & \left( \begin{array}{c} O_1 \\ O_2 \end{array} \right) \end{array}$$

$$S: \text{Diagram} = 0 \quad T: \text{Diagram} = 2 \quad G: \text{Diagram} = 0$$

$$NC: 2 \text{Diagram} = \text{Diagram} + \text{Diagram}$$

**Computable!**

$$\begin{array}{ccc} \text{via} & \text{Diagram} & \xrightarrow{\frac{1}{2} \text{Diagram}} \left[ \begin{array}{c} \text{Diagram} \\ -1 \\ \text{Diagram} \\ +1 \end{array} \right] & \xrightarrow{\frac{1}{2} \text{Diagram}} \text{Diagram} \end{array}$$

"complex simplification"

**Complexes:**

$$\Omega = (\Omega^{-n_-} \longrightarrow \Omega^{-n_-+1} \longrightarrow \dots \longrightarrow \Omega^{n_+})$$

**Morphisms:**

$$\begin{array}{ccccccc} \dots & \longrightarrow & \Omega_0^{r-1} & \xrightarrow{d^{r-1}} & \Omega_0^r & \xrightarrow{d^r} & \Omega_0^{r+1} \longrightarrow \dots \\ & & F^{r-1} \downarrow & & F^r \downarrow & & F^{r+1} \downarrow \\ \dots & \longrightarrow & \Omega_1^{r-1} & \xrightarrow{d^{r-1}} & \Omega_1^r & \xrightarrow{d^r} & \Omega_1^{r+1} \longrightarrow \dots \end{array}$$

**Homotopies:**

$$\begin{array}{ccccc} \Omega_0^{r-1} & \xrightarrow{d^{r-1}} & \Omega_0^r & \xrightarrow{d^r} & \Omega_0^{r+1} \\ F^{r-1} \parallel \downarrow & \nearrow h^{r-1} & F^r \parallel \downarrow & \nearrow h^r & F^{r+1} \parallel \downarrow \\ \Omega_1^{r-1} & \xrightarrow{d^{r-1}} & \Omega_1^r & \xrightarrow{d^r} & \Omega_1^{r+1} \end{array}$$

$$F^r - G^r = h^{r+1} d^r + d^{r-1} h^r$$

**The Main Point.** "The cube",  $\text{Kh}(L)$ , is an up-to-homotopy invariant of knots and links. Its Euler characteristic is the Jones polynomial, yet it is strictly stronger than the Jones polynomial. It is functorial (in the appropriate sense) and practically computable.

**The Categorification Speculative Paradigm.** • Every object in math is the Euler characteristic of a complex.  
 • Every operation lifts to an operation between complexes.  
 • Every identity remains true, up to homotopy.

All arrows in an arbitrary additive category!