

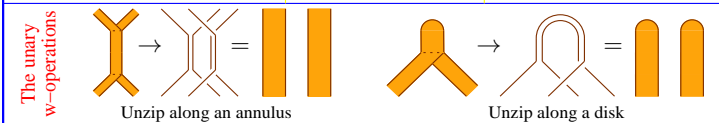
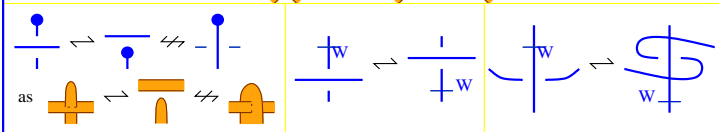
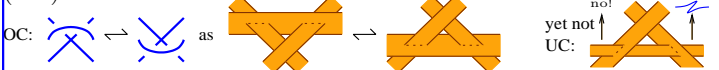
## Facts and Dreams About v-Knots and Etingof-Kazhdan, 2

### Example 6 - Ribbon 2-Knots.

Also, "movies of flying rings".



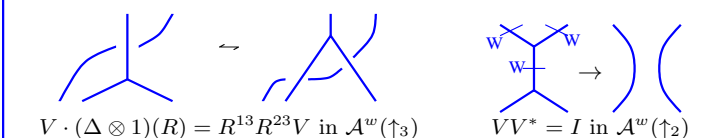
The  $w$ -relations include R234, VR1234, D, Overcrossings Commute (OC) but not UC:



### Trivalent $w$ -Tangles.

$$wTT = PA \left\langle \begin{array}{c|c|c} w\text{-generators} & w\text{-relations} & \text{unary } w\text{-operations} \end{array} \right\rangle = CA \left\langle \begin{array}{c} \text{same} \\ w/o \times \end{array} \right\rangle$$

**Theorem.** There exists a homomorphic expansion  $Z$  for  $wTT$ . In particular,  $Z$  respects  $R4$  and intertwines annulus and disk unzips:



**Alekseev-Torossian [AT]** (equivalent to Kashiwara-Vergne [KV]). There are elements  $F \in \text{TAut}_2$  and  $a \in \mathfrak{t}_1$  such that

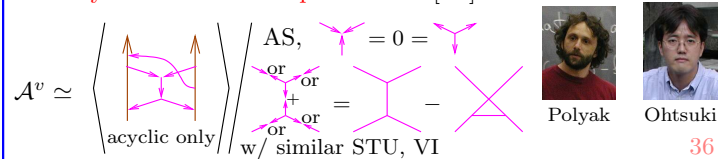
$$F(x+y) = \log e^x e^y \quad \text{and} \quad jF = a(x) + a(y) - a(\log e^x e^y).$$

**Theorem.** That's equivalent to a homomorphic expansion for  $wTT$ .

### The Main Example.

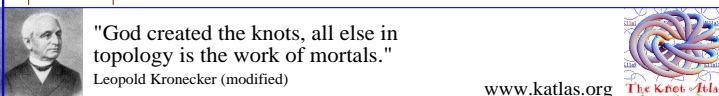
$$vTT = PA \left\langle \begin{array}{c|c|c} \text{generators} & \text{relations} & \text{unary operations} \end{array} \right\rangle = CA \left\langle \begin{array}{c} \text{same} \\ w/o \times \end{array} \right\rangle$$

### The Polyak-Ohtsuki Description of $\mathcal{A}^v$ [Po].



**$\mathcal{A}^v$  pairs with Lie bialgebras.** Let  $\mathfrak{g}_+$  be a Lie bialgebra with basis  $X_a$ , bracket  $[\cdot, \cdot]$ , cobracket  $\delta$ , dual  $\mathfrak{g}_- = \mathfrak{g}_+^*$ , dual basis  $X^a$  for  $\mathfrak{g}_-$ , double  $\mathfrak{g} = \mathfrak{g}_+ \oplus \mathfrak{g}_-$ , structure constants  $[X_a, X_b] = \sum c_{ab}^c X_c$  and co-structure constants  $\delta(X_a) = \sum c_{ab}^c X_b \otimes X_c$ . Then

$$\sum_{a,b,c,d,e,f=1}^{\dim \mathfrak{g}} b_{de}^c c_{ab}^c X_a X^d X_f \otimes X_b X^e X^f \in \mathcal{U}(\mathfrak{g})^{\otimes 2}$$



"God created the knots, all else in topology is the work of mortals." Leopold Kronecker (modified)

www.katlas.org



**Forbidden Theorem [EK, Ha, ?].** There exists a homomorphic expansion  $Z$  for  $vTT$ .

### Why Forbidden (to me)?

- Minor statement details may be off.
- No fully written proof.
- I don't understand the proof.
- There isn't yet a knot-theoretic view of the proof, like there is in the  $w$ -case.



Etingof

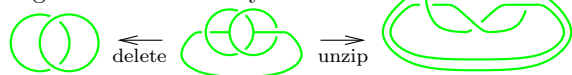
Haviv



Kazhdan

### Why Should We Care?

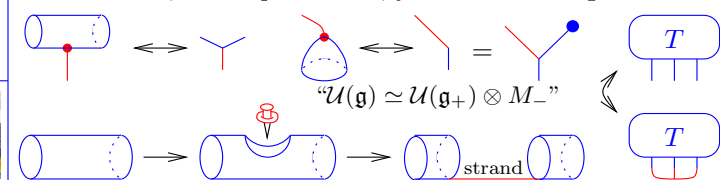
- A gateway into the forbidden territory of "quantum groups".
- Abstractly more pleasing: We study the things, and not just their representations.
- $\mathcal{A}^v$  is sometimes easier than  $\mathcal{A}^u$ : Alexander, say, arises easily from the 2D Lie algebra<sup>4</sup>.
- Potentially,  $\mathcal{A}^v$  has many more "internal quotients" than there are Lie bialgebras. What are they and what are the corresponding theories?
- My old<sup>5</sup> Algebraic Knot Theory dream:



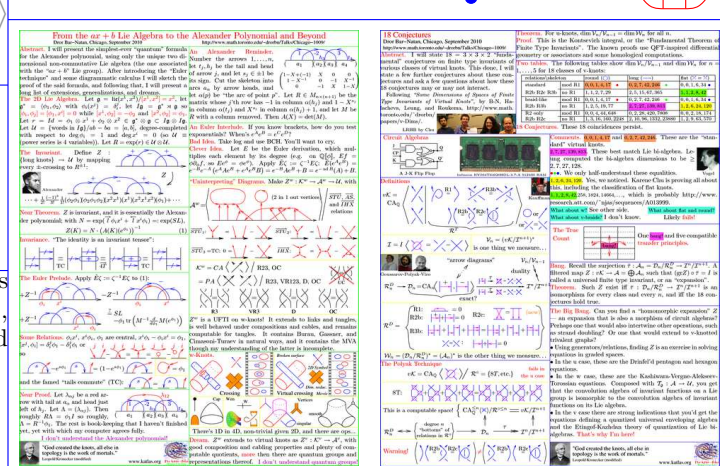
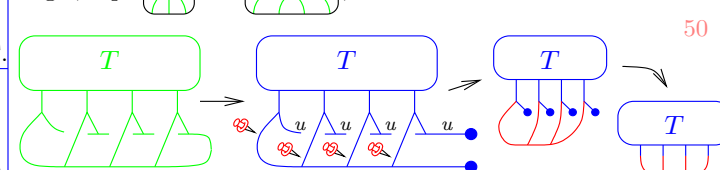
$V \rightarrow \Phi^{1\text{-loop}}$  after [AT]. "cut and cap" is well-defined(!) on  $\mathcal{K}^u$



$\Phi \rightarrow V$  after [AET]. In  $\mathcal{K}^w$  allow tubes and strand vertices, allow "punctures", yet allow no "tangles".



The generators of  $\mathcal{K}^w$  can be written in terms of the generators of  $\mathcal{K}^u$  (i.e., given  $\Phi$ , can write a formula for  $V$ ). With  $T$  any classical tangle, esp.  $\cap$  or  $\cup$ , consider the "sled"



Alexander is easy!

In Chicago, [BN4]

Many kinds of virtuals!

# Help Needed!