

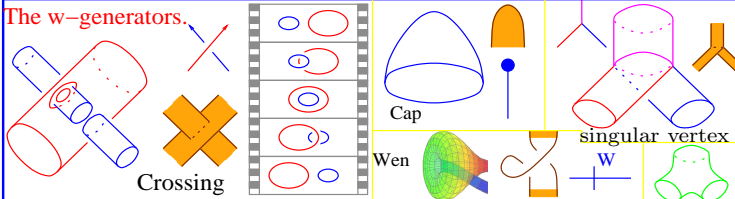
2. w-Knots, Alekseev–Torossian, and baby Etingof–Kazhdan

I understand Drinfel'd and Alekseev–Torossian, I don't understand Etingof–Kazhdan yet, and I'm clueless about Kontsevich
Dror Bar–Natan, Montpellier, June 2010, <http://www.math.toronto.edu/~drorbn/Talks/Montpellier-1006/>

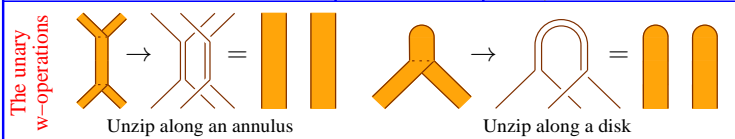
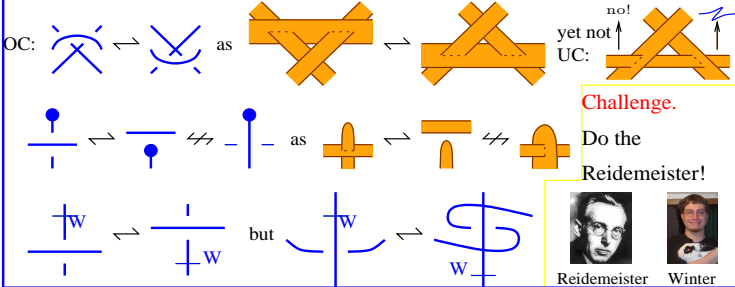
Trivalent w-Tangles.

$$wTT = CA \left\langle \begin{array}{c|c|c} \text{w-} & \text{w-} & \text{unary w-} \\ \text{generators} & \text{relations} & \text{operations} \end{array} \right\rangle$$

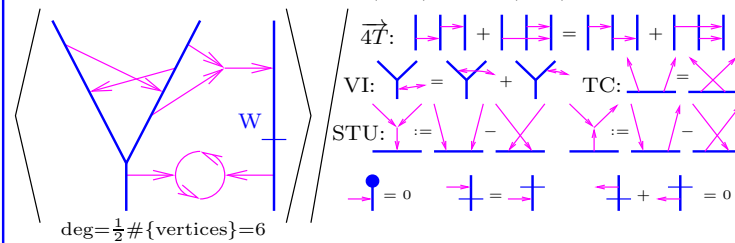
The w-generators.



The w-relations include R234, VR1234, D, Overcrossings Commute (OC) but not UC, $W^2 = 1$, and funny interactions between the wen and the cap and over- and under-crossings:



w-Jacobi diagrams and \mathcal{A} . $\mathcal{A}^w(Y \uparrow) \cong \mathcal{A}^w(\uparrow \uparrow \uparrow)$ is



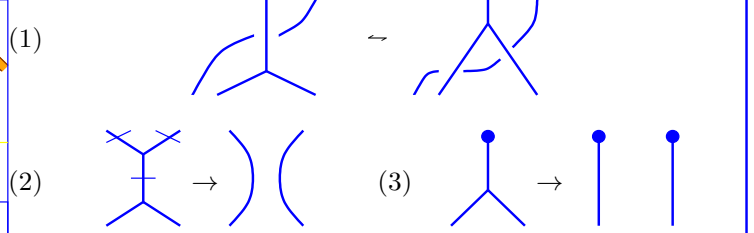
An Associator:

$$(AB)C \xrightarrow{\Phi \in \mathcal{U}(\mathfrak{g})^{\otimes 3}} A(BC)$$

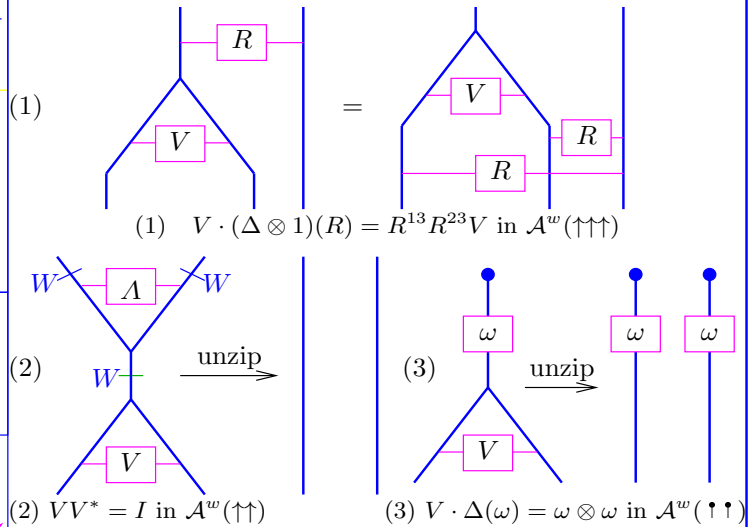
$$\Phi 1 \cdot (1\Delta 1) \Phi \cdot 1\Phi = (\Delta 11) \Phi \cdot (11\Delta) \Phi$$

The hexagon? Never heard of it.

Knot-Theoretic statement. There exists a homomorphic expansion Z for trivalent w-tangles. In particular, Z should respect $R4$ and intertwine annulus and disk unzips:



Diagrammatic statement. Let $R = \exp \uparrow \uparrow \in \mathcal{A}^w(\uparrow \uparrow)$. There exist $\omega \in \mathcal{A}^w(\uparrow)$ and $V \in \mathcal{A}^w(\uparrow \uparrow)$ so that



Alekseev–Torossian statement. There are elements $F' \in \text{TAut}_2$ and $a \in \mathfrak{tr}_1$ such that

$$F(x+y) = \log e^x e^y \quad \text{and} \quad jF = a(x) + a(y) - a(\log e^x e^y).$$

Theorem. The Alekseev–Torossian statement is equivalent to the knot-theoretic statement.

Proof. Write $V = e^c e^{uD}$ with $c \in \mathfrak{tr}_2$, $D \in \mathfrak{tdtr}_2$, and $\omega = e^b$ with $b \in \mathfrak{tr}_1$. Then (1) $\Leftrightarrow e^{uD}(x+y)e^{-uD} = \log e^x e^y$, (2) $\Leftrightarrow I = e^c e^{uD}(e^{uD})^* e^c = e^{2c} e^{jD}$, and (3) $\Leftrightarrow e^c e^{uD} e^{b(x+y)} = e^{b(x)+b(y)} \Leftrightarrow e^c e^{b(\log e^x e^y)} = e^{b(x)+b(y)} \Leftrightarrow c = b(x) + b(y) - b(\log e^x e^y)$.

The Alekseev–Torossian Correspondence.

$\{\text{Drinfel'd Associators}\} \Leftrightarrow \{\text{Solutions of KV}\}.$
We need an even bigger algebraic structure!
 $\left(\begin{array}{c} \text{green knotted trivalent} \\ \text{graphs in } \mathbb{R}^3 (u) \end{array} \right) \xrightarrow{\alpha_e} \left(\begin{array}{c} \text{blue tubes and red} \\ \text{strings in } \mathbb{R}^4 (\overline{w}) \end{array} \right)$

