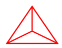
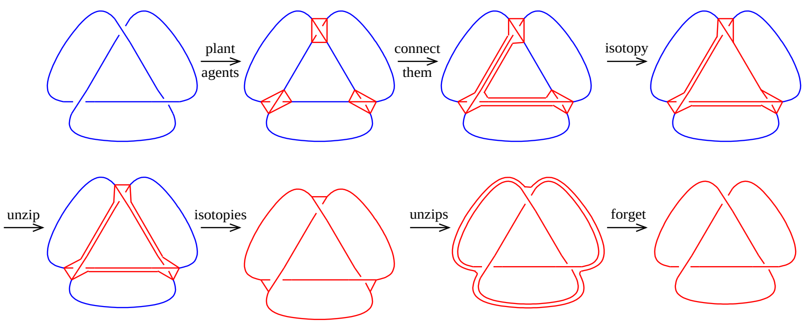
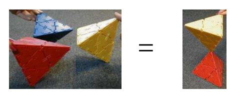
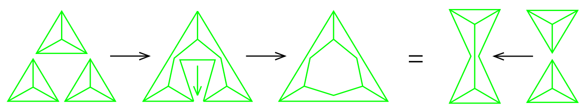
Using moves, KTG is generated by ribbon twists and the tetrahedron  :

All strands here are green

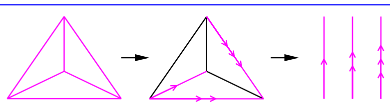


Modulo the relation(s): ()

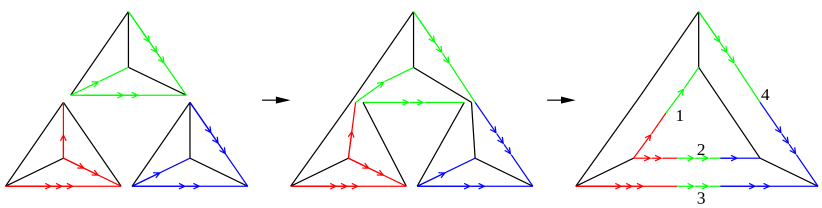


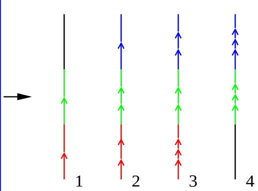
Claim. With $\Phi := Z(\text{tetrahedron})$, the above relation becomes equivalent to the Drinfel'd's pentagon of the theory of quasi-Hopf algebras.

Proof.

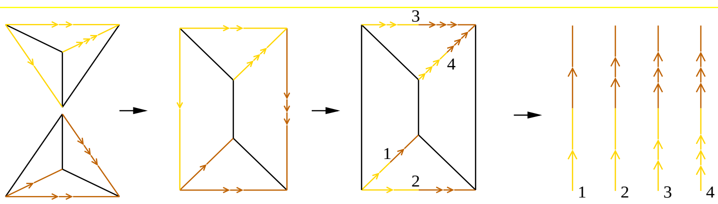


$:= \Phi \in \mathcal{A}(\uparrow_3)$



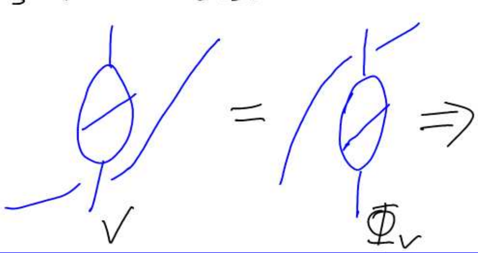


$= (\Phi \otimes 1) \cdot (1 \otimes \Delta \otimes 1)(\Phi) \cdot (1 \otimes \Phi) \in \mathcal{A}(\uparrow_4)$



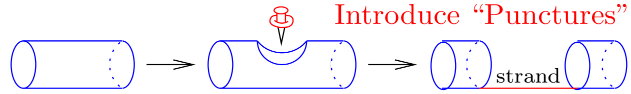
$= (\Delta \otimes 1 \otimes 1)(\Phi) \cdot (1 \otimes 1 \otimes \Delta)(\Phi) \in \mathcal{A}(\uparrow_4)$

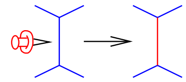
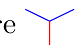
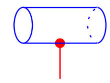
{SolKV} → {"Associators"}: Trivial – a tetrahedron has 4 vertices.

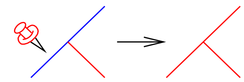


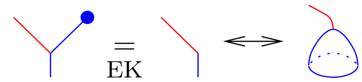
$\Phi_V \in$ Sder$_3$, so it is a sum of unrooted trees.

Introduce "Punctures".



So  where  means .

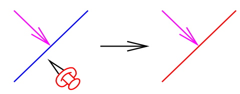
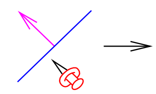
Punctures expand to the nearest Y-vertex: 

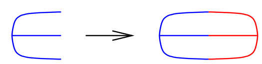
Note. 

$\mathcal{K}^{\overline{w}}$. Allow tubes and strands and tube-strand vertices as above, yet allow only "compact" knots — nothing runs to ∞.

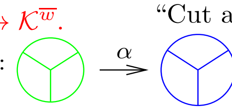
$\mathcal{K}^w \leftrightarrow \mathcal{K}^{\overline{w}}$ **equivalence.** $\mathcal{K}^w$ has a homomorphic expansion iff $\mathcal{K}^{\overline{w}}$ has a homomorphic expansion.

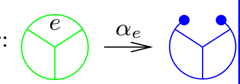
$\Longrightarrow$ Puncture $\mathcal{A}$ and Z:

 and  0.

$\Longleftarrow$  (makes sense, and CA ops can be emulated)

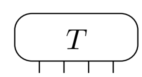
$\mathcal{K}^u \to \mathcal{K}^{\overline{w}}$. "Cut and cap is well-defined on u"

Light: 

Better: 

Theorem. The generators of $\mathcal{K}^{\overline{w}}$ can be written in terms of the generators of $\mathcal{K}^u$ (i.e., given Φ, can write a formula for V.

Sketch.

so enough to write any 

. Here go: