

Theorem. There exists a skeletal (very) planar algebra of "shielded tangles" with:

$$P_{ST}(\text{red/brown tangle}) = P_{ST}(\text{red/brown tangle})$$

(red and brown are always "knottable")

and with $(\text{red circle}, \text{brown circle})$

$$P_{ST}(\text{red/brown tangle}) = \text{red/brown tangle}$$

well defined!

and

$$P_{ST}(\text{red/brown tangle}) = (\text{red/brown tangle} \Rightarrow \text{red/brown tangle})$$

unzip

and

$$P_{ST}(\text{red/brown tangle}) = (\text{red/brown tangle} \Rightarrow \text{red/brown tangle})$$

bubble

Example.

$$\text{Example tangle} \text{ on } (\text{red circle}, \text{brown circle})$$

is

$$1. \text{ (red/brown tangle)} \rightarrow 2. \text{ (red/brown tangle)}$$

(so knottedness moved around!)

Facts. 1. There is no planar-algebra-structure-respecting universal Finite type invariant

$$\{\text{ordinary tangles}\} \rightarrow \langle \text{red/brown tangles} \rangle / \langle \text{4T, STU, AS, IHX} \rangle$$

2. But there is one for shielded tangles!

$$\exists \mathbb{Z} : \{\text{shielded tangles}\} \rightarrow \langle \text{red/brown tangles} \rangle / \langle \text{rels.} \rangle$$

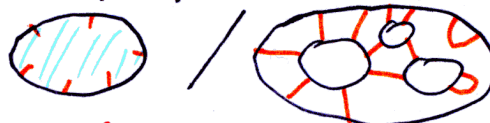
3. This $\mathbb{Z}$ provides a Reidemeister context for the Kontsevich integral!

4. A cousin of $\mathbb{Z}$ is equivalent to the Drinfeld theory of associators.

Dream-

A similar story will be told for "virtual knots", and will provide a topological interpretation of a "universal quantum group". See .../Talks/Hanoi-0708

Definition. A planar algebra has spaces / operations indexed by



(with obvious compatibility between ops.)

Examples. 1. My favourite - tangles:

$$P_T(\text{red/brown tangle}) = \langle \text{red/brown tangle} \rangle$$

makes Reidemeister's Theorem into gens/rels:

$$P_T = \langle \text{red/brown tangles} \rangle / \langle \text{rels.} \rangle$$

2. "skeletons":

$$S = P_T / \langle \text{rels.} \rangle = \{ \text{red/brown tangles} \}$$

Def. A skeletal planar algebra is "fibred" over S

3. TL:

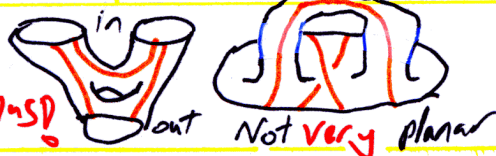
$$\{ \text{red/brown tangles} \} / \langle \text{rels.} \rangle = \{ \text{red/brown tangles} \}$$

4. Tensor: choose H , use

$$\{ \text{red/brown tangles} \} = H^{\otimes k}$$

appropriate contractions

All make sense in higher genus!



PROOF. key point:



on the level of skeletons - symmetric

$$\text{red/brown tangle} = \text{red/brown tangle}$$

unzip

... and trees are never knotted

1. Slides/beamers/Some propaganda powerpoint are evil!

*Can you always sync with the speaker?

*Don't you want to look back at pictures long gone?

2. Handouts are cool!

Everything's always in front of you, even when you go home.



"God created the knots, all else in topology is the work of mortals"
Leopold Kronecker (modified)



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