

# Dror's Dream Map of Quantum Groups $\subset$ Knot Theory

Drinfel'd, Etingof-Kazhdan

Prehistoric

## Hallucinations

about  
Khovanov

C A T E G O R I F I C A T I O N



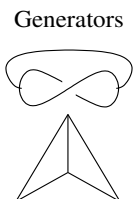
## Some harsh reality.

EK mix tangles and braids  
and algebras and Verma modules

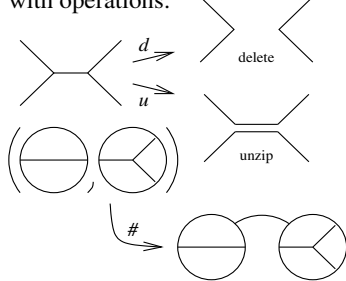
## Night Time Dreams

about  
Etingof - Kazhdan $\mathcal{K} / \text{translucent vertices}$  $\hat{\mathcal{K}} / tv$  $gr \hat{\mathcal{K}} / tv$  $\mathcal{A} / tv$  $\mathcal{K}_V = K(T)VG$ Knotted (trivalent?)  
virtual knots $\hat{\mathcal{K}}_V$  $gr \hat{\mathcal{K}}_V$  $\mathcal{A}$  $W$ specific  
quantum  
groups

## Day Time Dreams

about Reshetikhin  
- Turaev & quasi  
triangular quasi  
Hopf algebrasGens and rels?  $\Rightarrow$ Relations  
A 5 term relation,  
a 6 term relation,  
and lesser ones.

Definable subsets:

Genus  $g$  knots, ribbon knots, boundary links and more... $\mathcal{K} = KTG$   
Knotted Trivalent Graphs  
with operations: $\hat{\mathcal{K}}$ 

The unipotent completion

 $gr \hat{\mathcal{K}}$  $\mathcal{A}$ This is the Drinfel'd  
theory of Associators! $W_{sl(2)}$  $U(sl(2))$  $W_{sl(n)}$  $U(sl(n))$ 

etc.

## Why should we care?

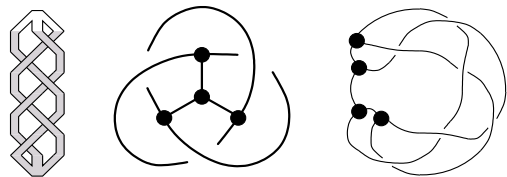
1. Usefulness!
2. Beauty!
3. Guidance!
4. Confidence!
5. Grothendieck-Teichmüller!



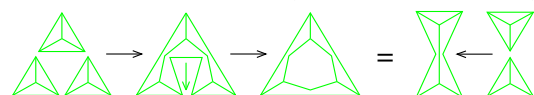
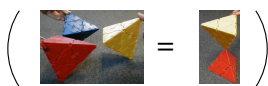
Fushimi-Inari

## A La Carte Drawings

Knotted Trivalent Graphs (KTG's):

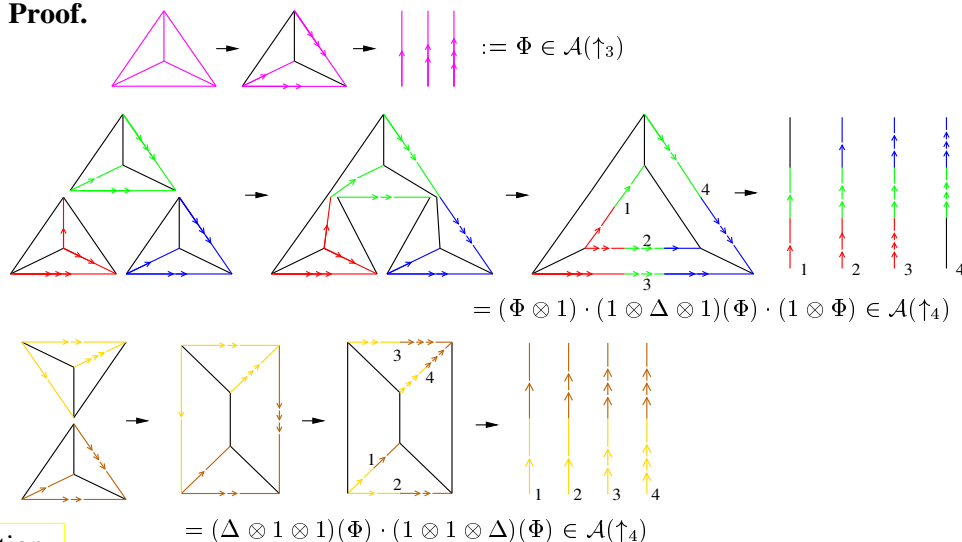


The 5-term relation:



**Claim.** With  $\Phi := Z(\Delta)$ , the above relation is equivalent to the Drinfel'd's pentagon equation.

## Proof.



Drinfel'd



Etingof



Kazhdan



J. Murakami



Ohtsuki



Kauffman



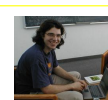
Goussarov



Polyak



Viro



D. Thurston



Haviv

More at <http://www.math.toronto.edu/~drorbn/Talks/Kyoto-0705/>