

The Kauffman Bracket: $\langle \emptyset \rangle = 1$; $\langle \bigcirc L \rangle = (q + q^{-1})\langle L \rangle$; $\langle \times \rangle = \langle \begin{smallmatrix} \diagup & \diagdown \\ \diagdown & \diagup \end{smallmatrix} \rangle - q \langle \begin{smallmatrix} \diagup & \diagup \\ \diagdown & \diagdown \end{smallmatrix} \rangle$.

The Jones Polynomial: $\hat{J}(L) = (-1)^{n_-} q^{n_+ - 2n_-} \langle L \rangle$, where (n_+, n_-) count $(\times, \times)$ crossings.

Khovanov's construction: $[[L]]$ — a chain complex of graded $\mathbb{Z}$ -modules;

$$[[\emptyset]] = 0 \rightarrow \underset{\text{height } 0}{\mathbb{Z}} \rightarrow 0; \quad [[\bigcirc L]] = V \otimes [[L]]; \quad [[\times]] = \text{Flatten} \left(0 \rightarrow \underset{\text{height } 0}{[[\begin{smallmatrix} \diagup & \diagdown \\ \diagdown & \diagup \end{smallmatrix}]]} \rightarrow \underset{\text{height } 1}{[[\begin{smallmatrix} \diagup & \diagup \\ \diagdown & \diagdown \end{smallmatrix}]]\{1\}} \rightarrow 0 \right);$$

$$\mathcal{H}(L) = \mathcal{H}(\mathcal{C}(L) = [[L]][-n_-]\{n_+ - 2n_-\})$$

$$V = \text{span}\langle v_+, v_- \rangle; \quad \deg v_{\pm} = \pm 1; \quad q\dim V = q + q^{-1} \quad \text{with} \quad q\dim \mathcal{O} := \sum_m q^m \dim \mathcal{O}_m;$$

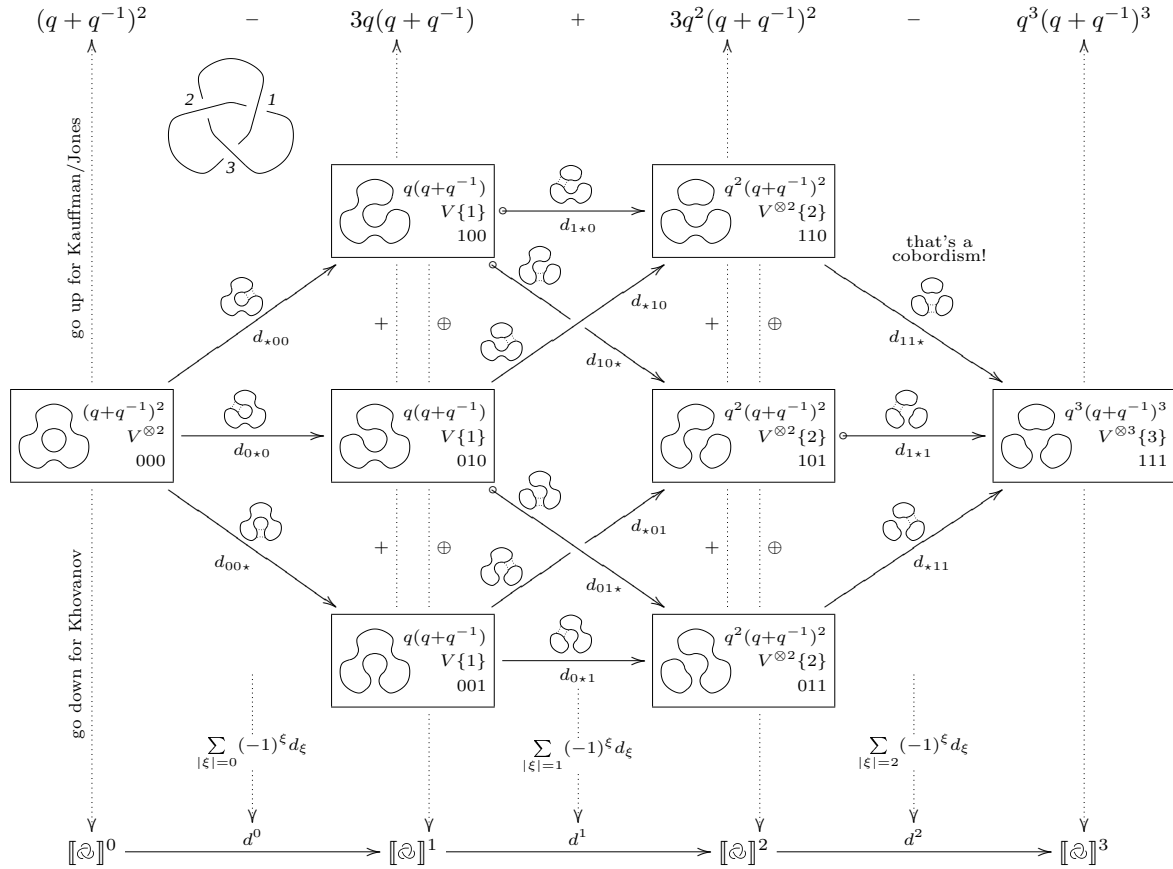
$$\mathcal{O}\{l\}_m := \mathcal{O}_{m-l} \quad \text{so} \quad q\dim \mathcal{O}\{l\} = q^l q\dim \mathcal{O}; \quad \cdot[s]: \quad \text{height shift by } s;$$

$$\begin{aligned} \left(\begin{array}{c} \bigcirc \quad \bigcirc \quad \xrightarrow{\quad} \quad \bigcirc \quad \bigcirc \end{array} \right) &\rightarrow (V \otimes V \xrightarrow{m} V) & m: \begin{cases} v_+ \otimes v_- \mapsto v_- & v_+ \otimes v_+ \mapsto v_+ \\ v_- \otimes v_+ \mapsto v_- & v_- \otimes v_- \mapsto 0 \end{cases} \\ \left(\begin{array}{c} \bigcirc \quad \bigcirc \quad \xrightarrow{\quad} \quad \bigcirc \quad \bigcirc \end{array} \right) &\rightarrow (V \xrightarrow{\Delta} V \otimes V) & \Delta: \begin{cases} v_+ \mapsto v_+ \otimes v_- + v_- \otimes v_+ \\ v_- \mapsto v_- \otimes v_- \end{cases} \end{aligned}$$

That's a Frobenius
Algebra! And a
(1+1)-dimensional
TQFT!

Example:

$$\curvearrowright \quad q^{-2} + 1 + q^2 - q^6 \xrightarrow[\text{(with } (n_+, n_-) = (3, 0))]{\cdot (-1)^{n_-} q^{n_+ - 2n_-}} q + q^3 + q^5 - q^9.$$



$$(\text{here } (-1)^\xi := (-1)^{\sum_{i < j} \xi_i} \text{ if } \xi_j = \star) \quad = \quad [[\bigcirc]] \xrightarrow[\text{(with } (n_+, n_-) = (3, 0))]{\cdot [-n_-]\{n_+ - 2n_-\}} \mathcal{C}(\bigcirc).$$

Theorem 1. The graded Euler characteristic of $\mathcal{C}(L)$ is $\hat{J}(L)$.

Theorem 2. The homology $\mathcal{H}(L)$ is a link invariant and thus so is $Kh_{\mathbb{F}}(L) := \sum_r t^r q\dim \mathcal{H}_{\mathbb{F}}^r(\mathcal{C}(L))$ over any field $\mathbb{F}$.

Theorem 3. $\mathcal{H}(\mathcal{C}(L))$ is strictly stronger than $\hat{J}(L)$: $\mathcal{H}(\mathcal{C}(\bar{5}_1)) \neq \mathcal{H}(\mathcal{C}(10_{132}))$ whereas $\hat{J}(\bar{5}_1) = \hat{J}(10_{132})$.

Conjecture 1. $Kh_{\mathbb{Q}}(L) = q^{s-1} (1 + q^2 + (1 + tq^4)Kh')$ and $Kh_{\mathbb{R}_2}(L) = q^{s-1} (1 + q^2) (1 + (1 + tq^2)Kh')$ for even $s = s(L)$ and non-negative-coefficients laurent polynomial $Kh' = Kh'(L)$.

Conjecture 2. For alternating knots s is the signature and Kh' depends only on tq^2 .

References. Khovanov's arXiv:math.QA/9908171 and arXiv:math.QA/0103190 and DBN's

<http://www.ma.huji.ac.il/~drorbn/papers/Categorification/>.

More at <http://www.math.toronto.edu/~drorbn/Talks/UW0-040213/>