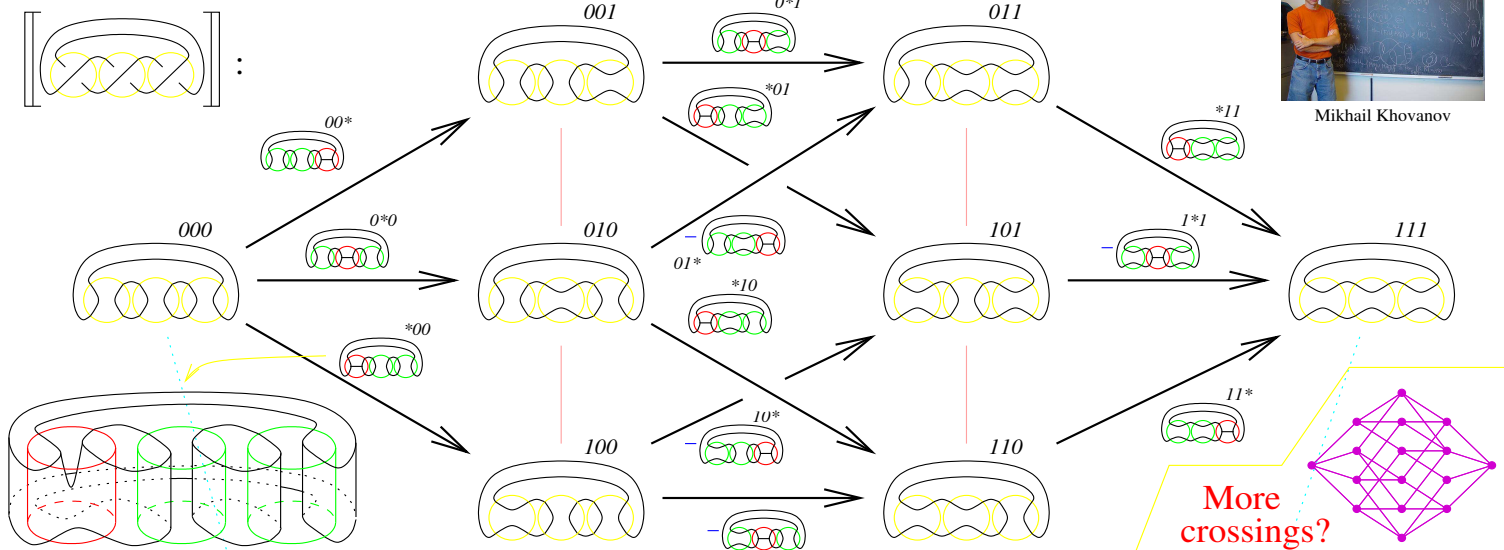


# Khovanov Homology



Mikhail Khovanov

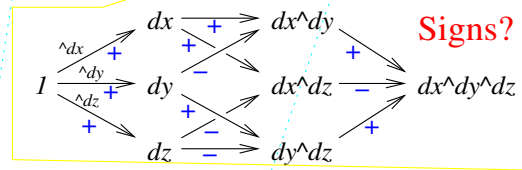


## What is it?

A cube for each knot/link projection;

Vertices: All fillings of with or with .

Edges: All fillings of  $I \times \text{Knot} = \text{Cobordism}$  with  $I \times \text{Crossing} = \text{Cobordism}$  or with  $I \times \text{Crossing} = \text{Cobordism}$  and precisely one .



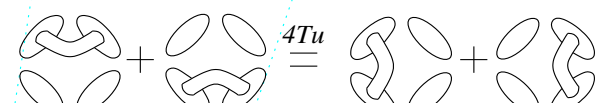
## Where does it live?

In  $Kom(Mat(\langle Cob \rangle / \{S, T, 4Tu\})) / homotopy$  :

$Kom$ : Complexes  $Cob$ : Cobordisms

$\langle \dots \rangle$ : Formal lin. comb.  $Mat$ : Matrices

$S$ : = 0  $T$ : = 2



## Jones/Kauffman?

A **TQFT** takes it to a complex whose graded Euler characteristic is the Jones polynomial.

The key point:  $\rightarrow V = \langle v_+, v_- \rangle$ ,  $\deg v_{\pm} = \pm 1$   
 $q\text{-dim } V = q + q^{-1}$

$$V^{\otimes 3} \longrightarrow (V^{\otimes 2} \oplus V^{\otimes 2} \oplus V^{\otimes 2})\{1\} \longrightarrow (V \oplus V \oplus V)\{2\} \longrightarrow V^{\otimes 2}\{3\}$$

$$(q + q^{-1})^3 \qquad 3q(q + q^{-1})^2 \qquad 3q^2(q + q^{-1}) \qquad q^3(q + q^{-1})^2$$



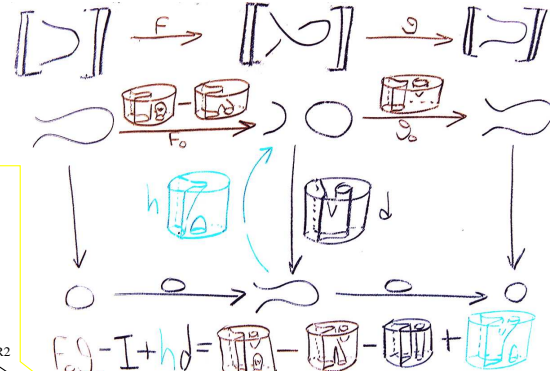
V. Jones



L. Kauffman

## But is it invariant?

(With similar proofs for R-II and R-III)



## Why is it interesting?

1. It is stronger than the Jones polynomial.
2. It is less understood than the Jones polynomial:
  - a. Does it have a topological interpretation?
  - b. Does it have a "physical" interpretation?
  - c. Does it also work for other quantum invariants?
  - d. Does it work for manifolds and for knots in manifolds?
  - e. Is there a relation with finite-type invariants?
  - f. Does it work for "virtual knots"?
3. Jacobsson, Khovanov: It is a functor!!!  
(from knots and cobordisms to complexes and morphisms)

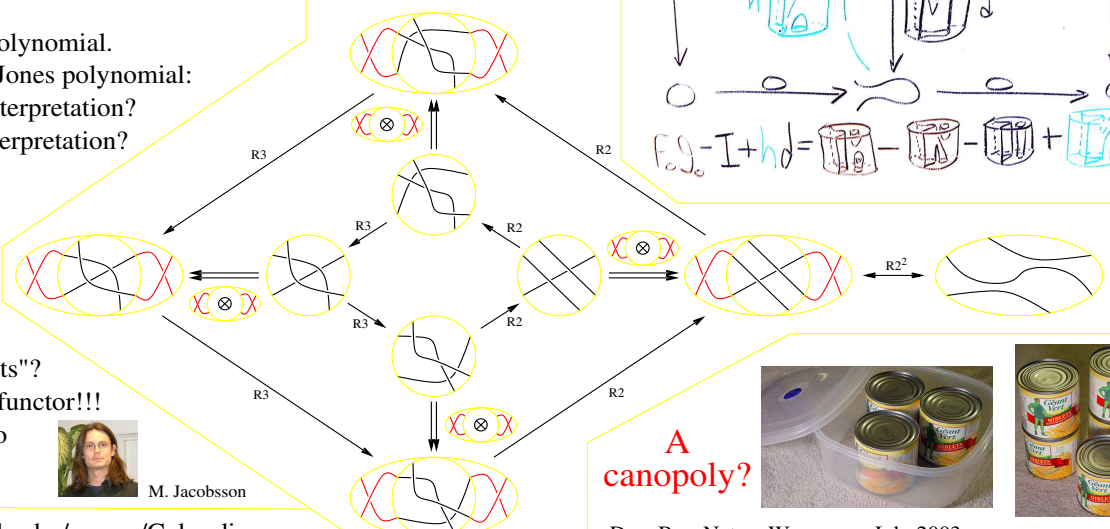


M. Jacobsson

See

<http://www.math.toronto.edu/~drorbn/papers/Cobordism>

## A functor?



## A canopoly?



Dror Bar-Natan, Warszawa, July 2003.

More at <http://www.math.toronto.edu/~drorbn/Talks/UW0-040213/>