

General Crossings

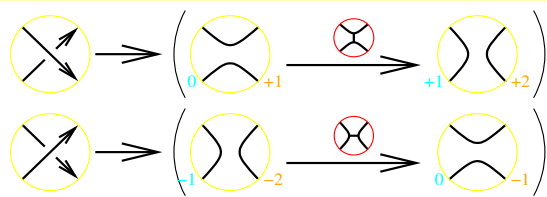
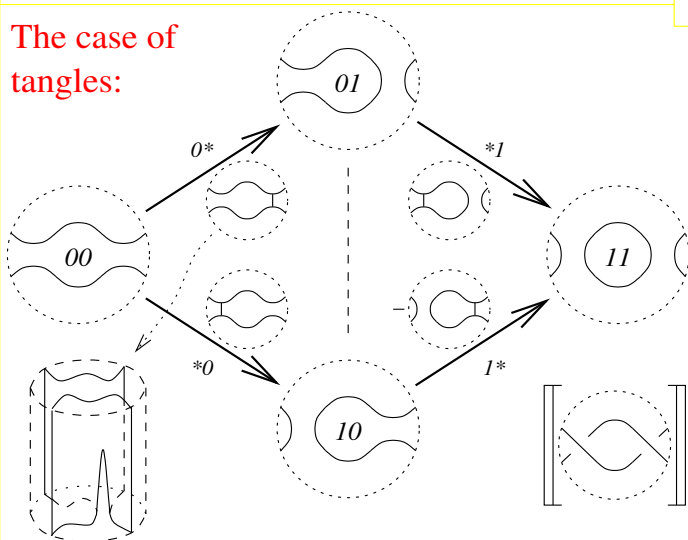


Figure 1 shows a diagrammatic equation for the S-matrix. On the left, the S-matrix is defined as the ratio of two diagrams: a sphere with a dashed line (labeled $S: = 0$) and a torus with a dashed line (labeled $= 2$). On the right, a diagrammatic equation is shown: a diagram of two linked loops plus a diagram of two separate loops equals $4Tu$ times a diagram of two linked loops plus a diagram of two separate loops.

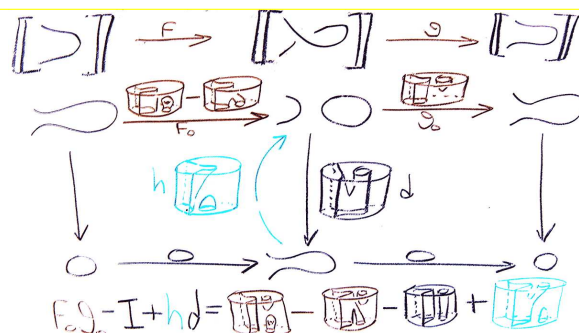
The case of tangles:



Invariant!



Kurt Reidemeister



The Reduction Lemma. If ϕ is an isomorphism then the complex

$$[C] \xrightarrow{\begin{pmatrix} \alpha \\ \beta \end{pmatrix}} \begin{bmatrix} b_1 \\ D \end{bmatrix} \xrightarrow{\begin{pmatrix} \phi & \delta \\ \gamma & \epsilon \end{pmatrix}} \begin{bmatrix} b_2 \\ E \end{bmatrix} \xrightarrow{(\mu \quad \nu)} [F]$$

is isomorphic to the (direct sum) complex

$$[C] \xrightarrow{\begin{pmatrix} 0 \\ \beta \end{pmatrix}} \begin{bmatrix} b_1 \\ D \end{bmatrix} \xrightarrow{\begin{pmatrix} \phi & 0 \\ 0 & \epsilon - \gamma\phi^{-1}\delta \end{pmatrix}} \begin{bmatrix} b_2 \\ E \end{bmatrix} \xrightarrow{\begin{pmatrix} 0 & \nu \end{pmatrix}} [F]$$

The work of Naot.

$\langle \text{surfaces} \rangle / 4\text{Tu}$ is freely generated by Shrek surfaces

A Shrek surface with 7 boundaries
(one distinguished), 3 handles and
2 tubes



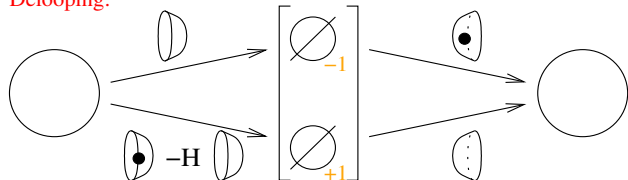
Gad Naot



שרעק

Let \bullet denote a tube to the distinguished component (the curtain), and let H denote a handle on the curtain. Then

Delooping:



... so the invariant is valued in complexes over a category with just one object and morphisms in $\mathbb{Z}[H]$; all is graded and $\deg H = -2$.

The work of Green.

standard data:

The universal invariant of the left-handed trefoil is



Jeremy Green

$$\begin{array}{c} \boxed{\text{Diagram}} \\ \xrightarrow{H} \end{array} \begin{array}{c} \text{Diagram} \end{array} \xrightarrow{0} \begin{array}{c} \text{Diagram} \end{array}$$

(and the invariant of the 48 crossing $T(8,7)$ is computable in minutes...)

Some functors.

$$\begin{array}{c}
 \text{H} \mapsto \begin{array}{c} \text{classical} \\ \begin{array}{ccc} \langle + \rangle & & \langle + \rangle \\ \langle - \rangle & \xrightarrow{2} & \langle - \rangle \end{array} \end{array} \\
 \text{Lee} \begin{array}{ccc} \langle + \rangle & \xrightarrow{2} & \langle + \rangle \\ \langle - \rangle & \xrightarrow{2} & \langle - \rangle \end{array}
 \end{array}
 \quad
 \begin{array}{c}
 \text{reduced} \\
 \langle 0 \rangle \xrightarrow{0} \langle 0 \rangle
 \end{array}
 \quad
 \begin{array}{c}
 \text{Lee} \\
 \begin{array}{ccc} \langle + \rangle & \xrightarrow{2} & \langle + \rangle \\ \langle - \rangle & \xrightarrow{2} & \langle - \rangle \end{array}
 \end{array}
 \quad
 \begin{array}{c}
 \frac{\mathbb{Z}[X]}{X^2 - hX - t} \\
 \begin{array}{ccc} o & i & - \\ + & -h & 2t \\ - & 2 & h \end{array}
 \end{array}
 \quad
 \begin{array}{c}
 ? \\
 \begin{array}{ccc} \langle 2 \rangle & \xrightarrow{\quad} & \langle 2 \rangle \\ \langle 0 \rangle & \xrightarrow{\quad} & \langle 0 \rangle \\ \langle -2 \rangle & \xrightarrow{\quad} & \langle -2 \rangle \end{array}
 \end{array}
 \end{array}$$

(Lee's spectral sequence and Rasmussen's invariant also recoverable)

<http://www.math.toronto.edu/~drorbn/Talks/UQAM-051001/>