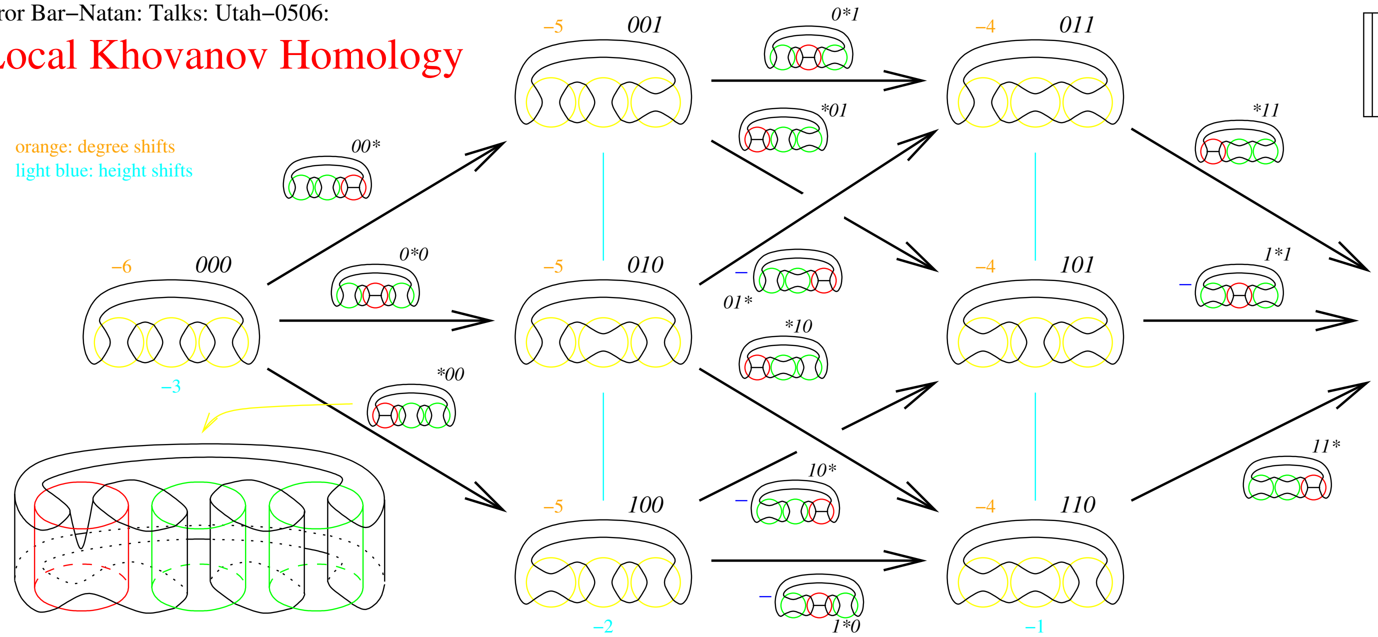
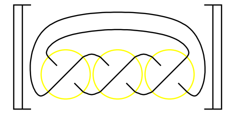
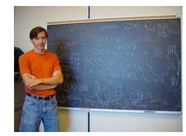
Dror Bar–Natan: Talks: Utah–0506:

Local Khovanov Homology

orange: degree shifts
light blue: height shifts



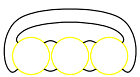
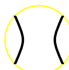
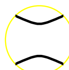


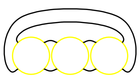
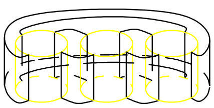
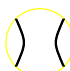
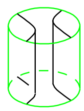
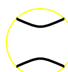
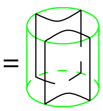
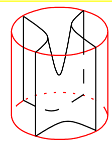


Mikhail Khovanov

What is it?

A cube for each knot/link projection;

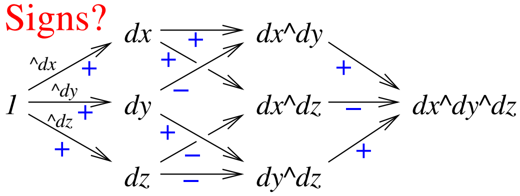
Vertices: All fillings of  with  or with  .

Edges: All fillings of $I\times$  =  with $I\times$  =  or with $I\times$  =  and precisely one  .

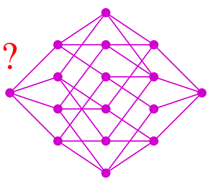
"God created the knots, all else in topology is the work of mortals"

Leopold Kronecker (modified)

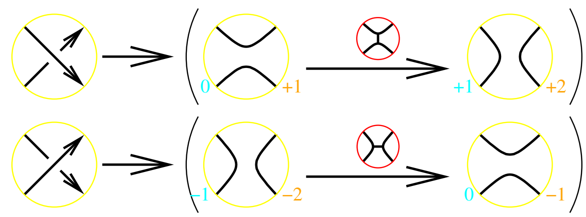
Signs?



More crossings?



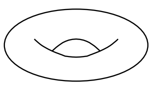
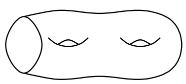
General Crossings

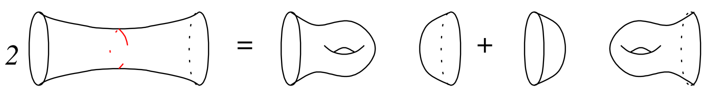


Where does it live?

In *Kom(Mat(<Cob> / {S, T, G, NC})) / homotopy*

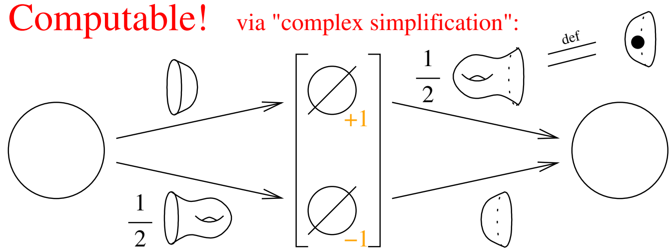
Kom: Complexes Mat: Matrices Cob: Cobordisms <...>: Formal lin. comb.

S:  *=0* *T:*  *=2* *G:*  *=0*

NC: 

Computable!

via "complex simplification":



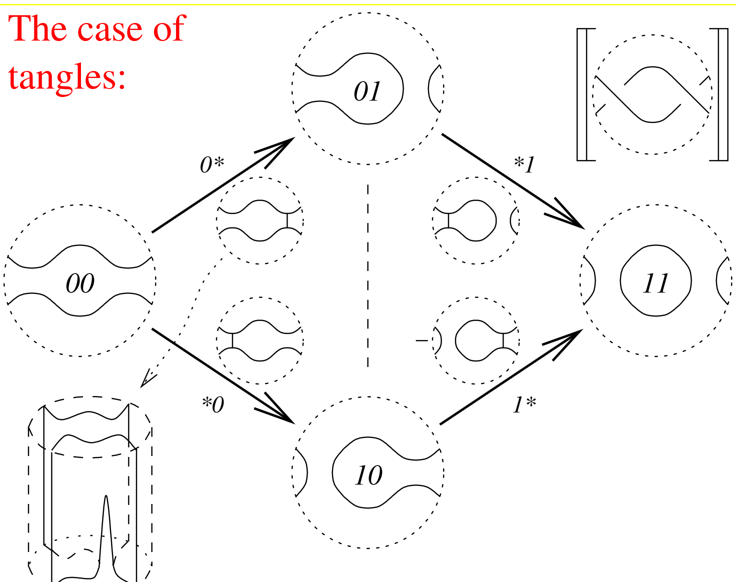
The Reduction Lemma. If ϕ is an isomorphism then the complex

$$[C] \xrightarrow{\begin{pmatrix}\alpha\\ \beta\end{pmatrix}} \begin{bmatrix} b_1\\ D\end{bmatrix} \xrightarrow{\begin{pmatrix}\phi & \delta\\ \gamma & \epsilon\end{pmatrix}} \begin{bmatrix} b_2\\ E\end{bmatrix} \xrightarrow{\begin{pmatrix}\mu & \nu\end{pmatrix}} [F]$$

is isomorphic to the (direct sum) complex

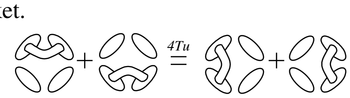
$$[C] \xrightarrow{\begin{pmatrix}0\\ \beta\end{pmatrix}} \begin{bmatrix} b_1\\ D\end{bmatrix} \xrightarrow{\begin{pmatrix}\phi & 0\\ 0 & \epsilon-\gamma\phi^{-1}\delta\end{pmatrix}} \begin{bmatrix} b_2\\ E\end{bmatrix} \xrightarrow{\begin{pmatrix}0 & \nu\end{pmatrix}} [F]$$

The case of tangles:



So what?

* Computable to 50–100 crossings!
* Extremely easy to prove invariance!
* A localized relation with Kauffman's bracket.
* Easily generalizes to surfaces, virtuals, etc.
* Better understanding of functoriality.
* Removing G and replacing NC with 4Tu yields a more general theory!
* Will shed light on mutation invariance.
* May shed light on Lee's theory.
* May shed light on Rasmussen's invariant.



http://www.math.toronto.edu/~drorbn/Talks/Utah–0506/