ALGEBRAIC TOPOLOGY, PROBLEM SET 1

1. Category theory language

Exercise 1.1. Prove that the fundamental group is a functor from the category of *pointed* topological spaces to the category groups.

Exercise 1.2. Let $\mathcal{C}$ be a category. If in the commutative diagram in $\mathcal{C}$

$$\begin{array}{ccccc} A_1 & \longrightarrow & A_2 & \longrightarrow & A_3 \\ \downarrow & & \downarrow & & \downarrow \\ B_1 & \longrightarrow & B_2 & \longrightarrow & B_3 \end{array}$$

- left and right squares are pushouts, then the outer square is a pushout;
- outer and left squares are pushouts, then the right square is a pushout;

2. Fundamental group: calculations and applications

Exercise 2.1. (1) Calculate the fundamental group of a 2-dimensional oriented surface with $n \geq 0$ handles. (*Hint:* recall from the first term how to "glue" such surfaces from a piece of paper and apply Van Kampen's theorem.)

Deduce that two surfaces with different amount of handles are not homeomorphic.

(2) Calculate the fundamental group of S^n, $n \geq 2$.

Exercise 2.2. Calculate the fundamental group of the "house with two rooms" from the Hatcher's book, page 4.

Exercise 2.3. Show that the closed half-plane is not homeomorphic to the whole plane.

3. An application of the Brouwer fixed point theorem

Exercise 3.1. Any 3×3 matrix with positive real entries has an eigenvector with positive coordinates.

Date: March 13, 2002.