ALGEBRAIC TOPOLOGY, PROBLEM SET 3

1. Fundamental group of knot complement

In the following exercises a knot is a piecewise linear or smooth embedding of S^1 into $\mathbb{R}^3$ or S^3.

Exercise 1.1. Let $K \subset S^3$ be a knot. Consider the image of K in $\mathbb{R}^3$ under the stereographic projection $\sigma : S^3 \setminus \{\mathrm{pt}\} \to \mathbb{R}^3$. Prove that $\pi_1(S^3 \setminus K, x_0) \cong \pi_1(\mathbb{R}^3 \setminus \sigma(K), \sigma(x_0))$.

Exercise 1.2. Compute the presentation of a fundamental group of the complement of a knot $K \subset \mathbb{R}^3$. (You may work along the lines of Exercise 22, p. 55 in the Hatcher's book).

Exercise 1.3. Compute, using the above exercise, a presentation of the trefoil knot (this is the $(3,2)$-toric knot) and find an isomorphism between the two different presentation of the fundamental group of the complement of the trefoil: as an abstract knot and as a toric knot ($< a, b \mid a^2b^3 >$). (*Hint:* It may be not easy as an algebraic problem, so use the picture!)

2. Covering maps. First examples.

Solve Exercises 1, 2, 3, 4 from page 79. In the Exercise 4 skip the second part (or solve it as an optional exercise, but first read Example 1.45, p. 77).

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