

Math 157 Analysis I — Term Exam 2

University of Toronto, December 2, 2002.

Name: _____ Student ID: _____

Solve the following 5 problems. Each is worth 20 points although they may have unequal difficulty. Write your answers in the space below the problems and on the front sides of the extra pages; use the back of the pages for scratch paper. Only work appearing on the front side of pages will be graded. Write your name and student number on each page. If you need more paper please ask the tutors. You have an hour and 50 minutes.

Allowed Material: Any calculating device that is not capable of displaying text.

Good Luck!

For Grading Use Only

1	/20
2	/20
3	/20
4	/20
5	/20
Total	/100

web version: <http://www.math.toronto.edu/~drorbn/classes/0203/157AnalysisI/TermExam2/Exam.html>

Name: _____ Student ID: _____

Problem 1. Prove that there is a real number x so that

$$x^{157} + \frac{157}{1 + x^2 + \cos^2 x} = 157.$$

If your proof uses the intermediate value theorem, state it clearly and prove that it follows from the postulate P13.

Name: _____

Student ID: _____

Problem 2.

1. Define in precise terms “ f is differentiable at a ”.
2. Let

$$f(x) = \begin{cases} x^2 & x \in \mathbb{Q}, \\ 0 & x \notin \mathbb{Q}. \end{cases}$$

Is f differentiable at 0? If you think it is, prove your assertion and compute $f'(0)$. Otherwise prove that it isn't.

Name: _____

Student ID: _____

Problem 3. Calculate dy/dx in each of the following cases. Your answer may be in terms of x , of y , or of both, but reduce it algebraically to a reasonably simple form. You do not need to specify the domain of definition.

(a) $x^3 + y^3 = 2$

(c) $y^4 + y^3 + xy = 1$

(b) $y = x/\sqrt{x^2 - 4}$

(d) $y = \sin(\sin(x))$

Name: _____ Student ID: _____

Problem 4.

1. Prove that if $f'(x) > 0$ on some interval then f is increasing on that interval.
2. Sketch the graph of the function $f(x) = x + \frac{4}{x^2}$.

Name: _____ Student ID: _____

Problem 5. Write a formula for $(f^{-1})''(x)$ in terms of f' , f'' and $f^{-1}(x)$. Under what conditions does your formula hold?

Name: _____

Student ID: _____

Name: _____

Student ID: _____

Name: _____

Student ID: _____