Dror Bar-Natan: Classes: 2004-05: Math 1300Y - Topology:

Agenda for September 28, 2004

Comment. f is continuous at x iff for every neighborhood V of $f(x)$, its inverse image $f^{-1}(V)$ *contains* a neighborhood of x.

Agenda. We will discuss two primary notions and the interaction between them and along the way also learn about sequences....

First notion — the product topology. (The naive definition and the box topology), definition by listing our requirements, uniqueness and existence, interaction with the trivial topology, the subspace topology, T_2 and the discrete topology.

Second notion — metric spaces and metrizability Definition, examples, the metric topology, T_2-ness, metrizability.

The interaction We'll prove three theorems:

Theorem 1. (good) $\emptyset \neq \prod_{k=1}^{\infty} X_k$ is metrizable iff every X_k is metrizable.

Theorem 2. (who cares?) $\mathbb{R}^{\mathbb{N}}_{\text{box}}$ is not metrizable.

Theorem 3. (bad) $\mathbb{R}^{\mathbb{R}}$ is not metrizable.

In order to prove Theorems 2 and 3 we will need to know about sequences, and these are quite interested by themselves:

Sequences. Convergence, sequential closure.

Proposition 1. The sequential closure is always a subset of the closure, and in a metrizable space, they are equal.

Proposition 2. If $F : X \to Y$ and X is metric, then f is continuous iff for every sequence in X, the convergence $x_k \to x$ implies the convergence $f(x_k) \to f(x)$.