Neutral fixed points and bifurcations.

1- Suppose that F has a neutral fixed point at x_0 with $F'(x_0) = 1$.

1. Suppose also that $F''(x_0) > 0$. What can you say about x_0: is it attracting, repelling, one-side attracting, one-side repelling?
2. Idem as in 5 but assuming now that $F''(x_0) < 0$.

2- Suppose that F has a neutral fixed point at x_0 with $F'(x_0) = 1$ and $F''(x_0) = 0$.

1. Suppose also that $F'''(x_0) > 0$. What can you say about x_0: is it attracting, repelling, weakly attracting, repelling?
2. Idem as before but assuming now that $F'''(x_0) < 0$.

3- Each of the following function has a neutral fixed point. Find this point and determine the type of it.

1. $F(x) = x + x^2$
2. $F(x) = x - x^2$
3. $F(x) = -x - x^2$
4. $F(x) = -x + x^2$
5. $F(x) = \frac{1}{x}$
6. $F(x) = \frac{-1}{2}x^3 - \frac{3}{2}x^2 + 1$
7. $F(x) = \exp(x - 1)$ (fixed point is $X_0 = 1$).
8. $F(x) = \sin(x)$
9. $F(x) = \tan(x)$
10. $F(x) = x + x^3$
11. $F(x) = x - x^3$
12. $F(x) = -x + x^3$

13. $F(x) = -x - x^3$
14. $F(x) = \log(|x-1|)$

4- Each of the following functions undergoes a bifurcation of fixed point at the given parameter value. In each case, identify the type of the bifurcation. In each case, identify the phase phase portrait of the bifurcation.

1. $F_\lambda(x) = x + x^2 + \lambda, \lambda = 0$
2. $F_\lambda(x) = x + x^2 + \lambda, \lambda = 1$
3. $F_\mu(x) = \mu x + x^3, \mu = 1$
4. $F_\mu(x) = \mu x + x^3, \mu = 1$
5. $F_\mu(x) = \mu \sin(x), \mu = 1$
6. $F_\mu(x) = \mu \sin(x), \mu = 1$
7. $F_c(x) = x^3 + c, c = \frac{2}{3\sqrt{3}}$
8. $F_\lambda(x) = \lambda(\exp(x)1), \lambda = 1$
9. $F_\lambda(x) = \lambda(exp(x)1), \lambda = 1$
10. $F_c(x) = cx^2 + x, c = 0$
11. $F_c(x) = x^3 + cx^2 + x, c = 0$

5- Consider the family $f_\mu(x) = x^3 + \frac{9}{2}x^2 + (5+\mu)x + \frac{1}{2}$ with μ close to 0.

1. Sketch the graph of f. Try to localize the fixed point of f_μ for $\mu = 0$.
2. Show that the family has only one neutral fixed point for $\mu = 0$. Is it attracting or repelling? Justify.
3. Study the phase portrait of the bifurcation of the neutral fixed point.

6- Let f_λ be the family $f_\lambda(x) = \lambda x - x^3$. Shows that there is a periodic point of period two for $\lambda > -1$. Is it repelling or attracting? Justify.

7- Considering the quadratic family $Q_c(x) = x^2 + c$

1. Prove that for $\frac{5}{4} < c < \frac{3}{4}$ there is an attracting periodic point of period two.
2. Prove that for $c = \frac{5}{4}$ there is a neutral periodic point of period two
3. Prove that for $c = \frac{5}{4}$ there is a repelling periodic point of period two.

7- Consider the quadratic family $F_\mu(x) = x^2 - \mu$ with $\mu \in [1, 9]$.

1. For each μ sketch the graph and find the fixed points. Are attracting or repelling? Justify.
2. For which values of μ there exists an invariant interval?
3. For which μ there are periodic point of arbitrarily large period? Justify.
4. For each μ find the set $\{x : F_\mu^n(x) \to +\infty\}$.
5. For each μ find the set $\{x : F_\mu^n(x) \to -\infty\}$.
6. For which parameter of μ there is a bifurcation?