A REMARKABLE THEOREM

by

Carl Friedrich Gauss

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Doctor of Philosophy
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University of Toronto

ABSTRACT

A Remarkable Theorem

Carl Friedrich Gauß

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Gauß' Theorema Egregium (Latin for "Remarkable Theorem") is a major result of differential geometry proved by Carl Friedrich Gauß that concerns the curvature of surfaces. The theorem is that Gaussian curvature can be determined entirely by measuring angles, distances and their rates on a surface, without reference to the particular manner in which the surface is embedded in the ambient 3-dimensional Euclidean space. In other words, the Gaussian curvature of a surface does not change if one bends the surface without stretching it. Thus the Gaussian curvature is an intrinsic invariant of a surface.

Gauß presented the theorem in this manner (translated from Latin):

Thus the formula of the preceding article leads itself to the remarkable Theorem. If a curved surface is developed upon any other surface whatever, the measure of curvature in each point remains unchanged.

The theorem is "remarkable" because the starting definition of Gaussian curvature makes direct use of position of the surface in space. So it is quite surprising that the result does not depend on its embedding in spite of all bending and twisting deformations undergone.

To someone,
who did something nice.

ACKNOWLEDGEMENTS

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PUBLICATIONS

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CONTENTS

1

DIFFERENTIABLE MANIFOLDS

In mathematics, a differentiable manifold[1] is a type of manifold that is locally similar enough to a linear space to allow one to do calculus. Any manifold can be described by a collection of charts, also known as an atlas. One may then apply ideas from calculus while working within the individual charts, since each chart lies within a linear space to which the usual rules of calculus apply.

1.1 TOPOLOGICAL MANIFOLDS

Here are two definitions.

Definition 1.1 (Locally Euclidean Space)**.** A topological space X is called *locally Euclidean* if there is a non-negative integer n such that every point in X has a neighbourhood which is homeomorphic to $\mathbb{R}^n$

Definition 1.2 (Topological Manifold)**.** A *topological manifold* is a locally Euclidean Hausdorff space.

1.2 DIFFERENTIABLE MANIFOLDS

Now let's define something else.

Definition 1.3 (Differentiable Manifold)**.** A *differentiable manifold* is a topological manifold equipped with an equivalence class of atlases whose transition maps are all differentiable.

1.3 MAIN THEOREM OF THIS THESIS

Theorem 1.4 (Theorema Egregium)**.** *The Gaussian curvature of a surface is invariant under local isometry.*

Proof. This is immediate from Definition 1.3. Details are left as an exercise to the reader. □

1 also differential manifold

2 EXAMPLES

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2.2 ANOTHER SECTION IN THIS CHAPTER

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2 De web nostre historia angloromanic.

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2.3 SOME MATH

We can define scalar multiplication in $\mathbb{R}^n$ by

$$c[u_1, \ldots, u_n] = [cu_1, \ldots, cu_n]$$

You can now check that for $u, v \in \mathbb{R}^n$, we have

$$c(u + v) = cu + cv$$

BIBLIOGRAPHY

[Gau27] Carl Friedrich Gauß, *General investigations of curved surfaces*, 1827.

COLOPHON

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