DEPARTMENT OF MATHEMATICS
University of Toronto

Complex Analysis Exam ($1\frac{1}{2}$ hour)

Monday, May 6, 2002, 1–2:30 p.m.

No aids.
Do all questions.
Questions will be weighted equally.

1. (a) Use the theory of residues to calculate the integral

$$\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$$

(b) What is the image of the unit disk in the w-plane under the mapping

$$w = z + \frac{1}{z}.$$

2. Let f be an analytic function in the punctured disk $\Delta = \{z : \ 0 < |z - z_0| < \Omega\}$ that has an essential singularity at z_0. Prove that $f(\Delta)$ is dense in $\mathbb{C}$.

3. Suppose that f is entire and satisfies $|f(z)| \le C|z|^k$ for all z, where C is a positive constant and k is a positive integer. What can you conclude about the form of f? Explain.