DEPARTMENT OF MATHEMATICS
University of Toronto

Complex Analysis Exam ($1\frac{1}{2}$ hours)

Tuesday, September 3, 2002, 1–2:30 p.m.

No aids.
Do all questions.
Questions will be weighted equally.

1. Use the theory of residues to evaluate

$$\int_0^\infty \frac{\cos x}{(x^2+b^2)^2}\,dx \qquad \text{where } b>0.$$

2. Let $f(z)$ be analytic for $|z|<R$. For $0 \le r < R$ let $M(r) = \sup\limits_{|z|=r} |f(z)|$.

Prove

a) that $M(r)$ is a continuous, non decreasing function of r.

b) If f is not constant then $M(r)$ is strictly increasing.

3. Let μ_n be a sequence of harmonic functions which converge uniformly on compact subsets of the open set $U \subset \mathbb{C}$ to a function μ. Prove that $\dfrac{\partial \mu_n}{\partial x}$ converges to $\dfrac{\partial \mu}{\partial x}$ uniformly on compact subsets of U.