

DEPARTMENT OF MATHEMATICS  
University of Toronto

**Complex Analysis Exam ( $1\frac{1}{2}$  hours)**

*Tuesday, September 2, 2003, 1–2:30 p.m.*

No aids.

Do all questions.

Questions will be weighted equally.

1. (a) State Schwarz's Lemma.

(b) Prove that every 1 - 1 analytic mapping from  $\Delta := \{z \mid |z| < 1\}$  onto  $\Delta$  is of the form

$$f(z) = e^{i\theta} \left( \frac{z - \alpha}{1 - \bar{\alpha}z} \right) \text{ for some } \alpha \in \Delta.$$

2. (a) Define normal family (of analytic functions) and state a general theorem which gives a criterion for a family of analytic functions to be normal.

(b) Consider  $\left\{ f \mid f = \sum_{n=0}^{\infty} a_n z^n \text{ with } |a_n| \leq n \text{ for } n = 1, 2, \dots \right\}$ .

Using (a) above (or otherwise) show that this is a normal family of analytic functions.

3. Let  $f$  be analytic on  $\Delta$  and 1 - 1 on  $\Delta^* := \{z \mid 0 < |z| < 1\}$ . Prove that  $f$  is 1 - 1 on  $\Delta$ .