DEPARTMENT OF MATHEMATICS
University of Toronto

Complex Analysis Exam ($1\frac{1}{2}$ hours)

Tuesday, September 2, 2003, 1–2:30 p.m.

No aids.
Do all questions.
Questions will be weighted equally.

1. (a) State Schwarz's Lemma.

(b) Prove that every 1 - 1 analytic mapping from $\Delta := \{z \mid |z| < 1\}$ onto Δ is of the form

$$f(z) = e^{i\theta}\left(\frac{z-\alpha}{1-\bar{\alpha}z}\right) \text{ for some } \alpha \in \Delta.$$

2. (a) Define normal family (of analytic functions) and state a general theorem which gives a criterion for a family of analytic functions to be normal.

(b) Consider $\left\{f \mid f = \sum_{n=0}^{\infty} a_n z^n \text{ with } |a_n| \leq n \text{ for } n = 1, 2, \ldots\right\}$.

Using (a) above (or otherwise) show that this is a normal family of analytic functions.

3. Let f be analytic on Δ and $1-1$ on $\Delta^* := \{z \mid 0 < |z| < 1\}$. Prove that f is $1-1$ on Δ.