DEPARTMENT OF MATHEMATICS
University of Toronto

Complex Analysis Exam ($1\frac{1}{2}$ hours)

Friday, May 7, 2004, 1–2:30 p.m.

No aids.
Do all questions.
Questions will be weighted equally.

1. (a) Define normal family of analytic functions.

(b) Let $\mathcal{F} = \{f \mid f \text{ is holomorphic in } |z| < 1 \text{ and } |f^{(n)}(0)| \le n! \text{ for } n = 0, 1, 2, \ldots\}$. Prove that $\mathcal{F}$ is a normal family.

2. (a) State Schwarz's Lemma.

(b) Let f be a 1–1 analytic mapping from the unit disc $\Delta = \{z \mid |z| < 1\}$ onto itself such that $f(0) = 0$. Prove that f is of the form $f(z) = e^{i\theta} z$ for some θ.

3. Let Ω_1, Ω_2 be connected open subsets of $\mathbb{C}$ and let $f\colon \Omega_1 \to \Omega_2$ be analytic. For v harmonic on Ω_2, prove that $v \circ f$ is harmonic on Ω_1.