University of Toronto

Department of Mathematics

Complex Analysis Examination

Monday, May 2, 2005, 1-2:30 p.m.
Duration 1 hour, 30 minutes

No aids allowed.
All questions are equal in value.

1. Let $\{f_n\}$ denote a sequence of holomorphic functions on a domain $\Omega \subset \mathbb{C}$, such that $\{f_n\}$ is bounded uniformly on compact subsets of Ω. Suppose that every subsequence $\{f_{n_k}\}$ which converges uniformly on compact sets, converges to the *same* holomorphic function f on Ω. Prove that $\{f_n\}$ converges to f uniformly on compact subsets of Ω.

2. Let f denote a holomorphic mapping of the unit disk to itself which in *not* the identity mapping. Show that f can have at most one fixed point.

3. Let f denote an entire function (which is not identically zero). Let n be a natural number. Prove that the following are equivalent:

   (a) There exists an entire function g such that $f = g^n$.

   (b) Every zero of f has order divisible by n.