University of Toronto
Department of Mathematics

Complex Analysis Examination

Tuesday, September 6, 2005, 1-2:30 p.m.
Duration 1 hour, 30 minutes

No aids allowed.
All questions are equal in value.

1. (a) Let U, V denote domains in $\mathbb{C}$, and let $f : U \to V$ be a holomorphic mapping. Suppose that f is *proper* (i.e., $f^{-1}(K)$ is compact, for every compact subset K of V). Prove that $f(U) = V$.

   (b) Is the assertion in (a) true if "holomorphic" is replaced by "continuous"? Explain.

2. Give an explicit description of the group of automorphisms of $\mathbb{C} \setminus \{0\}$ (i.e., the group of invertible holomorphic mappings from $\mathbb{C} \setminus \{0\}$ to itself).

3. Let $\{f_n\}$ be a uniformly bounded sequence of holomorphic functions on a domain $\Omega \subset \mathbb{C}$. Let $\{z_k\}$ be a sequence of distinct points in Ω with $\lim_{k\to\infty} z_k = z_0 \in \Omega$. Assume that $\lim_{n\to\infty} f_n(z_k)$ exists, for all k. Prove that $\{f_n\}$ converges uniformly on compact subsets of Ω.