University of Toronto
Department of Mathematics

Complex Analysis Examination

Tuesday, September 5, 2006, 1-2:30 p.m.
Duration 1 hour, 30 minutes

No aids allowed.
All questions are equal in value.

1. State each of the following carefully and precisely.

(a) The definition of an essential singularity as an isolated singularity of an analytic function.

(b) Liouville's Theorem.

(c) Laurent's Theorem about the Laurent series for a function analytic on an open annulus.

(d) Rouché's Theorem about the zeros of two analytic functions.

(e) The Little Picard Theorem.

2. Evaluate the following integrals by means of the residues

$$\text{(a)} \quad \int_{-\pi}^{\pi} \frac{\mathrm{d}\theta}{5 + 3\cos\theta} \qquad\qquad \text{(b)} \quad \int_{-\infty}^{\infty} \frac{1}{(x^2 + 4)^3}\mathrm{d}x.$$

3. What is the general form of a rational function (the quotient of two polynomials) which is of real values on the real axis? In particular, how are the zeros and poles located?