Department of Mathematics
University of Toronto

Complex Analysis Examination

September 2007
Duration 1 hour, 30 minutes

No aids allowed.
All questions are equal in value.

1. Does there exist a function $f(z)$ holomorphic in $\mathbb{C} \setminus \{0\}$, such that

$$|f(z)| \geq \frac{1}{\sqrt{|z|}} ,$$

   for all $z \in \mathbb{C} \setminus \{0\}$? Why?

2. Let f denote a holomorphic mapping of the unit disk to itself which is not the identity mapping. Show that f can have at most one fixed point.

3. Use residues to show that

$$\int_0^1 \frac{dx}{\sqrt[3]{x^2 - x^3}} = \frac{2\pi}{\sqrt{3}} .$$