

University of Toronto
Department of Mathematics
Complex Analysis PhD Qualifying Exam
1.5 hours
September 2011

No Aids

Questions will be weighted equally.

1. Construct a conformal mapping (as a composition of simpler maps) from the infinite strip $|\Im z| < \frac{\pi}{4}$ onto the unit disc which takes i to 0 . What is the most general such mapping ?

2. Suppose that Ω is a domain in \mathbb{C} , $\{f_k\}$ is a sequence of analytic functions on Ω , $f_k \rightarrow f$ uniformly on compact subsets of Ω , and f has a zero of order N at $z_0 \in \Omega$. Show that there exists $\rho > 0$ such that for k sufficiently large, $f_k(z)$ has exactly N zeros counting multiplicities on $|z - z_0| < \rho$.

3. Evaluate

$$\int_0^\infty \frac{\sin x}{x} dx$$

via residues. Justify your steps. Hint: Introduce e^{iz} and a suitable indented contour.