University of Toronto
Department of Mathematics
Complex Analysis PhD Qualifying Exam
September 5, 2012

No Aids
Time: 90 minutes
Questions will be weighted equally.

1. Consider the half-disc shaped domain $|z| < 1$, Im $z > 0$. Find the image of this domain under the transformation

$$w = \Big(\frac{z-1}{z+1}\Big)^2.$$

2. Let $a \in \mathbb{R}$, $a > 1$. Show that

$$ze^{a-z} = 1$$

has one solution with $|z| < 1$ and that it is real and positive.

3. Suppose that f is analytic in a domain G in the complex plane and not constant. Let D be a disc whose closure is contained in G. Suppose $|f|$ is constant on ∂D. Show that f has at least one zero in D.