DEPARTMENT OF MATHEMATICS
University of Toronto

**Topology Exam (3 hours)**

*May 1997*

No aids.
Do all questions.
All questions are of equal value.

1. Let $G$ be a group acting on a topological space $X$. $G$ acts *properly discontinuously* if every $x \in X$ has a neighbourhood $U_x$ such that $gU_x \cap U_x = \emptyset$ for all but finitely many $g \in G$. G acts *freely* if $gx = x$ implies $g = 1$ for any $x \in X$ and $g \in G$. Suppose $G$ is a group acting properly discontinously on a Hausdorff space $X$. Prove that if $G$ acts freely then the canonical map $\pi : X \to X/G$ is a covering map.

2. The *finite* topology on a set $X$ is the topology whose closed sets are $X$ and the finite subsets of $X$. Verify that $\mathbb{C}$ with the finite topology is not Hausdorff, is not second countable, but is separable. Show that a polynomial with complex coefficients defines a continuous map from $\mathbb{C}$ to itself.

3. Let $G$ be a path-connected topological group with identity element $e$. Prove that $\pi_1(G, e)$ is Abelian.

4. Calculate $H_*(\vee_k S^n, \mathbb{Z})$ for $k, n \geq 1$ .

5. For $n \geq 1$, what is the degree of the antipodal map on $S^n$ ? Give an example of a continuous map $f : S^n \to S^n$ of degree 2. Explain your answers.