DEPARTMENT OF MATHEMATICS
University of Toronto

Topology Exam (3 hours)

May 1997

No aids.
Do all questions.
All questions are of equal value.

1. Let G be a group acting on a topological space X. G acts *properly discontinuously* if every $x \in X$ has a neighbourhood U_x such that $gU_x \cap U_x = \emptyset$ for all but finitely many $g \in G$. G acts *freely* if $gx = x$ implies $g = 1$ for any $x \in X$ and $g \in G$. Suppose G is a group acting properly discontinously on a Hausdorff space X. Prove that if G acts freely then the canonical map $\pi : X \to X/G$ is a covering map.

2. The *finite* topology on a set X is the topology whose closed sets are X and the finite subsets of X. Verify that $\mathbb{C}$ with the finite topology is not Hausdorff, is not second countable, but is separable. Show that a polynomial with complex coefficients defines a continuous map from $\mathbb{C}$ to itself.

3. Let G be a path-connected topological group with identity element e. Prove that $\pi_1(G, e)$ is Abelian.

4. Calculate $H_*(\vee_k S^n, \mathbb{Z})$ for $k, n \geq 1$.

5. For $n \geq 1$, what is the degree of the antipodal map on S^n ? Give an example of a continuous map $f : S^n \to S^n$ of degree 2. Explain your answers.