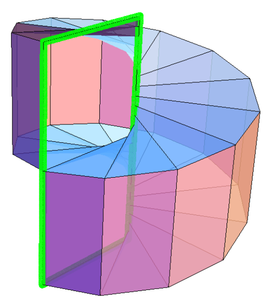
DEPARTMENT OF MATHEMATICS
University of Toronto

**Topology / Geometry Exam (3 hours)**

*Friday, September 7, 2012, 10AM — 1PM*

The 6 questions on the other side of this page have equal value, but different parts of a question may have different weights.



A Möbius Band

**Good Luck!**

**Problem 1.** (a) What is the homology of $S^1$? You need not prove this, just state the result.

(b) Explain what the Mayer-Vietoris sequence is. Use it to compute the homology of $S^n$ for all $n > 1$, assuming your result from (a).

**Problem 2.** (a) Use Stokes' theorem to compute the integral

$$\int_{S^2} x dy \wedge dz + y dz \wedge dx + z dx \wedge dy$$

Here

$$S^2 = \{x, y, z \in \mathbf{R}^3 : x^2 + y^2 + z^2 = 1\}.$$

(b) Let $\beta$ be the 2-form on $\mathbf{R}^3$ given by

$$\beta = -2y dx \wedge dy + 2z dx \wedge dz + dx \wedge dy - dy \wedge dz + yz dx \wedge dz + xz dy \wedge dz.$$

Is there a 1-form $\alpha$ on $\mathbf{R}^3$ satisfying $\beta = d\alpha$? Give a proof justifying your answer.

**Problem 3.** Show that if $g : S^2 \to S^2$ is continuous and $g(x) \neq g(-x)$ for every $x \in S^2$, then $g$ is surjective.

**Problem 4.** Let $M$ be a compact $n$-manifold and $N$ a connected $n$-manifold. Show that an embedding $f : M \to N$ must be surjective.

**Problem 5.** Compute the homology groups $H_*(X, A)$ where $X = S^1 \times S^1$ and $A$ is a finite set of points in $X$. You're allowed to take the homology of $X$ as known, yet if you use it, you should state what it is.

**Problem 6.** Let $X$ be the topological space obtained from $S^2$ by joining the north and the south poles by a straight line segment. Find the universal covering space of $X$. What is its group of covering transformations?

**Good Luck!**