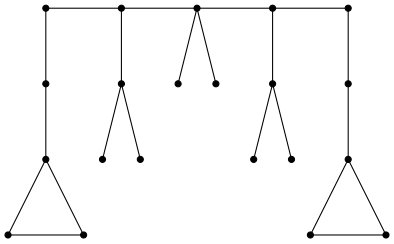
MAT332 - Fall 2016 - Homework 6

1. Calculate the chromatic polynomial of the graph below.



2. Let G be a simple graph with connected components $H_1, \ldots, H_c$.

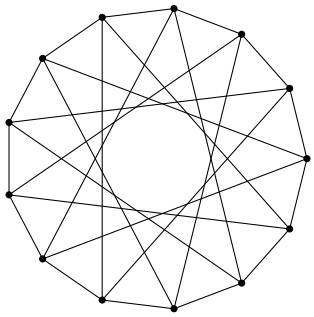
a. Prove that $\chi(G;k) = \chi(H_1;k)\chi(H_2;k)\ldots\chi(H_c;k)$.

b. Prove that the coefficient of k^ℓ in $\chi(G;k)$ is zero for all $\ell < c$.

3. Let C_n be the cycle graph on n vertices, $n \geq 3$. Prove that $\chi(C_n;k) = (k-1)^n + (-1)^n(k-1)$.

4. Prove that $R(a,b) \leq \binom{p+q-2}{p-1}$ for all $a, b \geq 2$. Hint: induction, and recall that $\binom{a}{b} = a!/(b!(a-b)!)$.

5. Use the fact that $R(3,4) = 9$ to prove that $R(3,5) = 14$. The graph below is a hint.



6 (review). Let G be a simple graph. Prove that G is bipartite if and only if G has no cycles of odd length.

7 (review). For a simple graph G, the **complement** of G, written $\overline{G}$, is the simple graph with $V(\overline{G}) = V(G)$, and two vertices are connected by an edge in $\overline{G}$ if they are *not* connected by an edge in G. Give a complete list of graphs G for which G and $\overline{G}$ are both trees. Explain how you are sure that there are no others.