

## **Proposal: I-AIM (Institute for Applied and Interdisciplinary Mathematics)**

### **1. Motivation and Goals: ('I aim' for excellence!)**

University of Toronto hosts the strongest collection of mathematical scientists in Canada. Apart from those appointed to the mathematics department, many others are dispersed across three campuses — in engineering, statistics, physics, chemistry, computer science and the biomedical and social sciences. There is also great potential for synergies by making contact with the many researchers whose work stands to benefit from a closer relationship to mathematics. To date links between such researchers have been formed mainly by serendipity, and few resources have been dedicated to nurturing these essential links, or to orienting students excited by the possibilities for mathematics in other disciplines.

We plan to found an elite research institution (I-AIM) that will function as umbrella organization to coordinate and benefit the common interests of these scholars and students. The emphasis will be on the discovery and application of new mathematics, models, and computational methods, driven by and for science. The institute will be staffed via cross-appointments between existing faculty coupled with dynamic new appointments in vital areas. It will also coordinate the training of students and postdoctoral fellows who are as well-versed in mathematics as in some area of application. This will be done through a new interdisciplinary program of graduate studies designed to facilitate joint mentoring by scientists and mathematicians working in different disciplines.

I-AIM is interdisciplinary by nature, and will focus on the long-term development of scientific programs at the University of Toronto. This complements the Fields Institute, a six-university consortium which hosts revolving one-year programs of a topical nature, selected to balance the needs of the regional, national, and international communities in pure and applied mathematics.

Our concrete goals include the following:

- To create an environment of unsurpassed quality for interdisciplinary research, which leads to the discovery and development of fundamental mathematics, new science, and transformative technology, from the crucible of their mutual interaction.
- To unite abstract mathematical expertise with specific domain knowledge: number theory with cryptography; probability theory with data mining; statistics with finance; partial differential equations with computer vision, new materials, and climate modelling; dynamical systems with pattern formation; geometry with string theory; harmonic analysis with signal processing, inverse problems with medical imaging, computer science with genomics, proteomics, molecular and evolutionary biology; numerical analysis with large-scale simulation.
- To build upon recent hiring in applied mathematics both within the math department, and in joint-appointments with other departments (ECE, CS, PHYS, STAT, SGS), while creating the infrastructure and working conditions to attract world-class mathematical scientists, theoreticians and practitioners in other disciplines.
- To develop an interdisciplinary graduate program in applied mathematics modelled on the successful programs at Princeton, Chicago, and the Courant Institute.
- To showcase interdisciplinary work already taking place on campus to students and the international community, aiding recruitment of top student and postdoctoral talent.
- To create a space for interdisciplinary interactions to occur, including a home on campus for applied mathematics as well as for the program in mathematical finance.
- To provide a natural base for the nationally distributed laboratory in Computational Sciences under consideration by the NRC, thus encouraging NRC to maximize its investment in Toronto.

## 2. Structure of I-AIM

The institute will consist of a core of Organizing Members, together with a larger group of Affiliated Faculty. Their activities within the institute will focus around the cultivation and strengthening of partnerships through research collaboration, the organization of interdisciplinary seminars, and the recruitment and training of jointly supervised graduate students and postdoctoral fellows. We see the development of the institute in three phases, outlined below. Administratively, the Institute will naturally be linked to the Department of Mathematics, with its Director reporting to the Chair. However, to fulfill its interdisciplinary mandate, I-AIM will also have an Advisory Board, consisting of representatives from the Affiliated Faculty, who will advise the Director and serve as liaisons between their home departments and I-AIM. To maintain global standards of excellence, I-AIM will also be advised by an appointed External Board.

**Phase 1 — Consolidation.** The first task of I-AIM is to consolidate existing strength across campus in the mathematical sciences and related disciplines. This effort will not yield returns unless a focal point can be provided, including adequate space to house I-AIM and its activities: weekly seminars, research projects, HQP. This will include reworking the graduate program in applied mathematics, with special attention to recruitment, course offerings, and program requirements. Suitable course offerings throughout the University will be incorporated into the Institute program, while some Mathematics Department offerings will be adapted for accessibility and relevance to cognate disciplines. Here there may be opportunities for economy when similar course offerings by different units can be eliminated, relieving pressures on teaching and resulting in a net gain in faculty research time. At this stage of development, I-AIM will require a Director to implement the recommendations of the advisory councils (internal and external), to report to the Mathematics Department, to liaise with the University, funding agencies (CIAR, MITACS, NCE, NSERC, NRC, the Fields Institute) and industry, and to coordinate the institute activities and postdoctoral program with the various departments and units. It will also require a Graduate Coordinator to oversee and actualize the revision of the curriculum in applied mathematics; to coordinate with the graduate directors of the cognate departments, and to provide academic advice to interdisciplinary graduate students from the departments which choose to participate. An administrative assistant will be necessary to assist the Director and Graduate Coordinator with their responsibilities, to assist students and postdoctoral fellows, and to provide all feasible assistance to Affiliated Faculty with their initiatives in applied mathematics.

**Phase 2 — Growth.** The lifeblood of any research institute is a constant infusion of new ideas. For I-AIM, this means having a steady stream of top-quality postdoctoral talent, distinguished long-term sabbatical visitors, and dynamic interdisciplinary speakers passing through the institute, and contributing to its research and teaching missions. In addition to faculty grant and departmentally sponsored postdoctoral fellows leveraged by Institute funds, we propose to start a program of prestigious named assistant professorships, which will have a CLTA status within the university, but enjoy reduced teaching load and conditions favorable enough to attract the best international talent. At least one of these three-year positions would be offered each year, and coordinated where possible between mathematics and a second host department. With this postdoctoral program in place, it becomes feasible to broaden the mission of the institute to include undergraduate training in applied and interdisciplinary mathematics. The Institute will then require an Undergraduate Coordinator to advise students and undertake a curriculum revision to the undergraduate program in applied mathematics similar to the revisions to the graduate program described above.

**Phase 3 — Global Leadership.** To achieve a position of global leadership in interdisciplinary mathematics, new appointments are necessary to fill in critical gaps in our base of expertise, bind together existing strengths and open new directions on our research agenda. Five of these

are described below — several joint between mathematics and another discipline. Furthermore, the NRC is currently considering a proposal to create a nationwide laboratory in Computational Science, consisting of up to 100 positions distributed across the country. The proposed institute will complement the NRC initiative, create a natural base for additional NRC positions, and encourage NRC to maximize their investment in the University of Toronto.

### 3. Sample Research Areas

Among the sciences, mathematics is distinguished by its formal nature and deductive methodology. It is a language and a mode of thought; it seeks to distill the essence of a single problem or a range of related phenomena, and to analyze the underlying universal structure using logic, models, and computation. Results of pure mathematics laid the groundwork for many of the triumphs of the last century — such as general relativity, quantum mechanics, game theory, and computing machines — which in turn guided development of the next generation mathematics. But to make progress on the hard and complex problems of the modern technological world, mathematicians must also work closely with experts in other domains. I-AIM brings together top researchers and students from the University and around the world to attack big problems with complementary methodologies: scientific experiment, mathematical modelling and proof, computer simulation.

**(A) Communications and Control:** (*Blake, Brumer, Broucke, Colliander, Francis, Feuerwerker, Jurdjevic, Kschischang, Lidar, Lo, Maggiore, Mann, Murty, Nachman, Penn, Sargent, Sarris, Tall, Valaee*). The digital age heralds a revolution in communications technology, serving vital commercial and national interests. With these technologies come new challenges: data encryption for security, verification, and privacy; protocols for error detection and correction; fighting dispersion and dissipation to maximize transmitted information with minimal resources. The possibility of quantum computing changes the rules of the game, turning established protocols on their heads. I-AIM will provide a natural framework for coordinating activities of number theorists from the GANITA laboratory, computer scientists specializing in cryptography, and the Communications group in ECE, while bringing expertise on nonlinear waves and solitons within the mathematics department into contact with the design problems facing engineers. Signal and image processing represents another set of questions, where nonlinear partial differential equations combine with neural networks to address challenges in machine intelligence. The Quantum Information and Quantum Control initiative would be a valuable participant in I-AIM.

**(B) Nonlinear Dynamics:** (*Brumer, Colliander, Kapral, Khesin, Ivrii, Jerrard, McCann, Mitrovica, Morris, Murray, Peltier, Pugh, Schofield, Shepherd, Sulem, Shub*). Nonlinear dynamics play a crucial role in scientific modelling, from nanoscale to biosphere, and from tabletop experiment to cosmological scale. There is a natural synergy of mathematicians studying partial differential equations, dynamical systems, and Hamiltonian mechanics, with scientists using these equations to model the atmosphere and oceans, geological and environmental processes, pattern formation, reaction chemistry, population biology, and dynamical cosmology. Toronto already boasts great strength arrayed around these areas, but a gap separates cultures between mathematics and the sciences. I-AIM is the vehicle for bridging this gap. An appointment in nonlinear dynamics spanning interests between mathematics, physics, CITA and chemistry is critical for consolidating existing strengths into a cohesive interdisciplinary group.

**(C) String Theory, Gravitation and Cosmology:** (*Bond, Hori, Peet, Pen, Poppitz, Khesin, Jeffrey*). String theory, gravitation and cosmology address the most fundamental questions about the underlying structure of the physical universe. As far as answers to these questions are understood, they reduce surprisingly often to geometry and symmetry. Thus mass is curvature, motion is geodesic, and particles correspond to excitation of vibrations with different symmetries in the

fabric of our universe. Mathematics — particularly geometry — has a significant role to play in understanding these issues. Recent spectacular progress includes the mathematical proofs of the Riemannian Penrose inequalities, which show how the surface area of black holes bounds the attraction they exert far away; the cosmic censorship conjecture, which states that singularities in the fabric of spacetime can almost surely be detected only by observers who are permanently incommunicado; and the stability of empty space to gravitational ripples. New challenges include understanding the connections between local and global notions of mass, the large scale geometry and topology of the universe, detecting gravity waves, and deriving testable predictions of string theory. We propose a new position in geometric analysis, to fill a vacuum between the recent joint appointments in string theory, strong mathematical groups in nonlinear partial differential equations and symplectic geometry, and the gravitational expertise in Astronomy and Astrophysics and CITA.

**(D) Discrete Math: Algorithms, Complexity, Geometry, Statistical Mechanics.** (*Borodin, Molloy, Rackoff, Shub, Virag, Whittington*). Fundamental problems in computer science with widespread ramifications revolve around the discovery of fast algorithms or proofs that they do not exist. These are intimately connected to measuring the complexity of tasks, finding clever geometrical and graph theoretic constructions, and accounting for the limiting behaviour of these structures as the task size increases without bound. Similar problems arise in statistical mechanics, where the manifested behaviour of macroscopic systems is well described by the infinite particle number limit. The discontinuities realized in this limit are called “phase transitions” and familiar in every day life. The same mathematics underlies both phenomena, accounting for the successes of probabilistic methods of proof, Monte Carlo simulation, and renormalization group methods in both areas. I-AIM will provide a venue for the cross-fertilization between theoretical chemistry and theoretical computer science, with discrete mathematics, combinatorics, geometry and statistical mechanics stimulating both sides. An appointment in statistical mechanics and discrete mathematics is key to the consolidation of strength.

**(E) Finance, Probability and Statistics, Data Mining:** (*Almgren, Bond, Christara, Feuerverger, Jackson, Kramer, Knight, Quastel, Rosenthal, Seco*). Uncertainty is an essential aspect of the world. As theoretical tools for modeling randomness developed, and large data sets began to permit quantitative analyses, probabilistic and statistical techniques moved to the fore in many areas of science. Finance is the paradigmatic example because of its importance in society, but the same problems — modelling and managing randomness, coping with enormous data sets, implementing large-scale and distributed computation — recur in engineering, bioinformatics, economics, and the natural sciences. I-AIM presents an opportunity for addressing these problems on a united front, sharing experiences acquired in the different disciplines to carve out better solutions and new discoveries. The central parts of mathematics involved are analysis, probability and statistics, areas in which we have a young and active group built with recent hires, and scientific computation — an area which needs further development.

**(F) Scientific Computation and Modelling:** (*Almgren, Bond, Buchweitz, Christara, Enright, Groth, Jackson, Peltier, Pen, Pugh, Repka, Schofield, Sulem, Whittington, Zingg*). Scientific computation and numerical simulation provide a new methodology in science and engineering whose importance can scarcely be overestimated. Current groups on campus employ this methodology to design everything from molecules to aeroplanes and telescopes, and to model the spreading of disease, environmental contaminants, climate change, the evolution of matter in the universe. I-AIM presents an opportunity for technical and intellectual exchange between these groups, which share different goals but a common methodology. The Mathematics Department includes a smattering of this expertise, but much of it lies elsewhere. With powerful computers, mathematical modelling, analysis, and simulation acquire a new relevance, mandating growth within the Department in

this direction. Much of this increase may come through the distributed laboratory in mathematical computation proposed by the NRC, but it is essential for Toronto that I-AIM be positioned to maximize this NRC investment. Since the Mathematics Department needs to consolidate its representation in computational sciences and modelling in any case, it is essential to prepare for the NRC initiative by creating at least one new position in scientific computation and another in modelling.

**(G) Biology and Biomedical Research:** (*Emili, Feuerverger, Friesen, Joy, Lewis, Nachman, Pugh, Repka, Schofield, Shub, Steipe, Tall, Tillier, Tropak, Wodak*). Biology and Biomedical research are areas of emerging potential for mathematical contribution. One focus of research is medical imaging technology: reconstructing 3-d images from noisy two dimensional slices, solving inverse problems, matching problems, feature identification and segmentation. In addition, vast quantities of epidemiological, genomic, and protein folding data, demand models, organizing principles, and data mining approaches. These are addressed by the biostatistics and bioinformatics groups. Computational biology and computational neuroscience are booming areas of research with computer scientists, physicists, mathematicians, and neurologists working in concert. The challenge is often to translate and define biological problems to make them accessible to rigorous analysis. The theoreticians who succeed in doing so have enormous impact. I-AIM will be positioned to respond nimbly to developments in these and other emerging areas. It will provide a forum for biological, biomedical, and mathematical researchers to search out common ground.

#### 4. Resources Needed

**Phase 1 (Consolidation).** The institute needs a space to call its own, including seminar rooms, a lounge, administrative offices, laboratory space for interdisciplinary projects and office space for jointly sponsored postdoctoral fellows and graduate students. Within the mathematics department, space has become a crisis, and there is little space to spare in cognate disciplines. A commitment of resources comparable to the 6th floor of the Bahen center would at the same time allow us to unite the group of applied mathematics currently divided between 22 Russell Street and 100 St. George, bring the mathematical finance program back onto campus and supply the necessary laboratory space for interdisciplinary graduate students, postdoctoral fellows, and sabbatical visitors. Proximity to the Fields' Institute, CHEM, CS, ECE, PHYS, and the future Department of Mathematics at 215 Huron Street are advantages. Teaching reduction of 1 FCE each will be provided to the Director of I-AIM and to the Graduate Coordinator by the Mathematics Department. Salary for an administrative assistant will need to be provided by the University. The availability of university matching funds for faculty contributions are necessary to make attractive graduate student and postdoctoral offers. Funding for a high profile interdisciplinary seminar series (\$25,000 /year) .

**Phase 2 (Growth).** CLTA lines for three named assistant professorships to be phased in: 3 x \$ 60,000 / year, plus sabbatical visitors 3 x \$40,000/ year. Teaching release of 1 FCE to be provided to the Undergraduate coordinator, either by the Mathematics Department or any cognate Department which desires to be involved in the administration of the institute (also as Graduate Coordinator or Director).

#### **Phase 3 (Global leadership in interdisciplinary mathematical sciences)**

The following new positions fill in critical gaps in our base of expertise, binding together existing strengths and opening new directions on our research agenda. We propose them as joint or cross appointments between MATH and at least one other department:

1. nonlinear dynamics (linked to physics, CITA or chemistry)
2. geometric analysis (ideally linking to CITA, astronomy, string theory)
3. discrete math and statistical mechanics (linked to CS, statistics, chemistry or physics)

4. scientific computation (linked as appropriate)
5. mathematical modelling (materials science, biological, biomedical, environmental)

## Appendix A:

Discussions to define the full scope and shape of I-AIM have only begun, and many people have yet to be contacted. However, the following faculty members have already expressed strong support for the proposal:

Robert Almgren, Computer Science and Mathematics; Mathematical Finance Program  
Ian Blake, Electrical and Computer Engineering  
Richard Bond, Canadian Institute of Theoretical Astrophysics  
Allan Borodin, Computer Science  
Mireille Broucke, Electrical and Computer Engineering  
Paul Brumer, Chemistry  
Ragnar Buchweitz, Mathematics  
James Colliander, Mathematics  
Wayne Enright, Computer Science  
Kentaro Hori, Mathematics and Physics  
Victor Ivrii, Mathematics  
Jim Friesen, Banting and Best Department of Medical Research  
Ken Jackson, Computer Science  
Robert Jerrard, Mathematics  
Mike Joy, Institute of Biomaterials and Biomedical Engineering  
Velimir Jurdjevic, Mathematics  
Raymond Kapral, Chemistry  
Boris Khesin, Mathematics  
Daniel Lidar, Chemistry  
Hoi-Kwong Lo, Electrical and Computer Engineering and Physics  
Manfredi Maggiore, Electrical and Computer Engineering  
Peter Martin, Astronomy and Canadian Institute for Theoretical Astrophysics  
Robert McCann, Mathematics  
Jerry Mitrovica, Physics  
Michael Molloy, Computer Science  
Stephen Morris, Physics  
Norman Murray, Canadian Institute of Theoretical Astrophysics  
Kumar Murty, Mathematics  
Adrian Nachman, Electrical and Computer Engineering and Mathematics  
Joe Repka, Mathematics  
Amanda Peet, Physics  
Richard Peltier, Physics  
Gerald Penn, Computer Science  
Ue-Li Pen, Canadian Institute for Theoretical Astrophysics  
Erich Poppitz, Physics  
Mary Pugh, Mathematics  
Jeremy Quastel, Mathematics and Statistics  
Jeffrey Rosenthal, Statistics  
Jeremy Schofield, Chemistry  
Luis Seco, Mathematics  
Theodore Shepherd, Physics  
Michael Shub, Mathematics  
Boris Steipe, Biochemistry, Molecular & Medical Genetics; Proteomics & Bioinformatics Program  
Catherine Sulem, Mathematics

Elisabeth Tillier, Medical Biophysics  
Balint Virag, Mathematics and Statistics  
Stuart Whittington, Chemistry

**Contact person:** Robert McCann, (416) 978-4658, [mccann@math.utoronto.ca](mailto:mccann@math.utoronto.ca)



## Appendix B: (Updated April 21, 2004)

Further expressions of interest and support from faculty members received following the initial submission of this proposal on January 30, 2004 as part of the provostial planning process:

Mireille Broucke, Electrical and Computer Engineering

Christina Christara, Computer Science

Al-Amin Dhirani, Chemistry

Andrew Emili, Medical Research, Medical Genetics & Microbiology; Proteomics & Bioinformatics

Andrey Feuerverger, Statistics

Bruce Francis, Electrical and Computer Engineering

Clinton Groth, Institute for Aerospace Studies

David Hogg, Medicine and Medical Biophysics

Keith Knight, Statistics

Lisa Kramer, Rotman School of Management

Frank Kschischang, Electrical and Computer Engineering

Peter Lewis, Biochemistry and Medicine

Steve Mann, Electrical and Computer Engineering

Jerry Mitrovica, Physics

Gerald Penn, Computer Science

Charles Rackoff, Computer Science

Joe Repka, Mathematics

Ted Sargent, Electrical and Computer Engineering

Costas Sarris, Electrical and Computer Engineering

Jeremy Schofield, Chemistry

Frank Tall, Mathematics

Michael Tropak, Hospital for Sick Kids

Shahrokh Valaee, Electrical and Computer Engineering

Shoshana Wodak, Biochemistry, Structural Biology, and Medical Genetics

David Zingg, Institute for Aerospace Studies