Problem 1

Which of the following operators are linear?

1. $L(u) = \sin x u(x, y) - \cos x$.
2. $L(u) = u_{xy} - e^x u$.
3. $L(u) = u'' + e^x (u')^2 - u$.

Problem 2

Are the polynomials x, $1 + x$, $x^2 - 2x$ linearly dependent or independent? Do they span the space of all polynomials of degree at most two?

Problem 3

Find the general solution of the equation

$$u' - 2xu = -2x.$$

Problem 4

Verify that the function $u(x, t) = e^{-k\lambda^2 t} \sin(\lambda x)$ satisfies the diffusion equation

$$u_t = k u_{xx}.$$

Problem 5

Find the function $u(x, y)$ satisfying the equation

$$\frac{\partial u(x, y)}{\partial x} = y u(x, y),$$

and the condition $u(0, y) = y$.

Due date: September 20, 2012