Problem 1

Consider the problem

$$u''(x) + u(x) = f(x), \quad u(0) = u(\pi) = 0,$$

where f(x) is a given function.

Does this equation have a solution for every function f? If the equation has a solution for a given f, is it unique?

Problem 2

Find the general solution of the equation

$$u_{xx} + 3u_{xt} + 2u_{tt} = 0$$

Hint: Use the procedure we used to solve the Wave equation.

Problem 3

Find the solution of the Wave equation

 $u_{tt} = u_{xx}$

with the initial condition $u(x,0) = u_t(x,0) = e^{2x}$.

Problem 4

Show that for a solution of the equation

$$u_{tt} = c^2 u_{xx} - u_t$$

the energy is a decreasing function of t. **Hint:** repeat the calculations for the derivative of the energy.

Problem 5

Find the maximum and the minimum of a solution of the diffusion equation, $u(x,t) = x^2 - 3x + 2kt$, in the square

$$\{(x,t): 1 \le x \le 2, 1 \le t \le 2\}.$$

Due date: October 4, 2012